Property Rights Enforcement with Unverifiable Incomes*

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Abstract

I study the extent of secure property rights a planner can implement. Agents can produce output, appropriate others' output, or work in property rights enforcement. The planner pays enforcement personnel using taxes collected from producers who can hide income from taxation at a cost. The planner implements perfectly secure property rights by incentivizing production through redistributive taxation and absorbing potential appropriators as enforcement personnel. Both taxation and employment in enforcement institutionalize redistribution that would otherwise take place through appropriation. Higher costs of hiding income permit more redistributive taxation and less enforcement, leading to more production and higher welfare.

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1 Introduction

Secure property rights matter for economic outcomes (see, e.g., Knack and Keefer (1995), Hall and Jones (1999), Acemoglu and Johnson (2005)). A society looking to protect property rights needs to commit designated resources. The availability of means to hide income from taxation constrains society in its effort to collect these resources from its members. In the U.S., for example, the Internal Revenue Service estimates that, from 2008 to 2010, on average more than 16 percent of the total tax liability remained unpaid after enforced and late payments; and that tax underreporting accounted for 85 percent of the gap before enforced payments.[1] This complication in raising funds may affect society's choice of a property rights enforcement regime. Can it by itself help explain imperfectly secure property rights and differences across countries? Absent other complications, like political economy frictions or agency problems, what can a planner achieve when agents can hide income from taxation?

I address these questions in an environment related to Murphy et al. (1993) and Acemoglu (1995). There are two types of agents with different productivities. Each agent can produce output, appropriate others' output, or work in enforcement. Those who produce output acquire property rights over it—the right to consume it, and the right to exclude others from consuming it. Protecting these property rights requires the planner to employ enforcement personnel to counter appropriation activities. In the context of this paper, appropriation activities are, for example, rent-seeking, corruption, theft and property crimes, fraud, or extortion; and they do not target particular groups nor require any special skill.[2] In the first-best outcome both appropriation and enforcement are absent and property rights are perfectly secure. However, a planner can neither force agents into an occupation nor verify the true incomes of agents she wants to tax. Producers can misrepresent their income by hiding some of it at a cost so as to be subject to a different tax payment.

Although this informational friction constrains the planner, she does not tolerate any appropriation. She always achieves an outcome with perfectly secure property rights by employing enforcement personnel financed by redistributive taxation. This policy resembles the link between institutional investment and redistribution emphasized by Koeppl et al. (2014) for efficient contract enforcement in a production economy.[3] Here, the planner may use subsidies to encourage unproductive agents to abstain from appropriation and instead contribute to the pie available to society. She uses the sector for enforcement to absorb all those who

[1]See Internal Revenue Service (2016). An estimate of the contribution of underreporting to the gap after enforced payments is not available. Tax compliance between 2008 and 2010 was comparable to that in 2006 (see pp. 6 and 8). See, e.g., Andreoni et al. (1998) on tax underpayment and evasion more generally.

[2]Tullock (1967), Rose-Ackerman (1975), and Becker (1968) started large literatures on some such activities. The availability of these activities affects the allocation of talent and resources. See, e.g., Baumol (1990), Murphy et al. (1991, 1993), Acemoglu (1995), and Acemoglu and Verdier (1998).

[3]On inequality, redistribution, and crime, see, e.g., Benoît and Osborne (1995), İmrohoroğlu et al. (2000).

do not produce and might otherwise engage in appropriation. Taxation and employment in enforcement institutionalize redistribution, replacing appropriation. Higher costs of hiding income from taxation permit more income redistribution through taxation. A steeper tax schedule induces more agents to produce and allows the planner to employ fewer enforcement personnel. More agents producing more output increases aggregate consumption and welfare.

These results arise for three reasons. First, property rights are enforced by personnel whose wellbeing the planner values. Second, more taxation maps directly into more enforcement. While more taxation decreases the payoffs from both production and appropriation, more enforcement makes production more profitable relative to appropriation. At the same time, the planner pays enforcement personnel a fixed wage that needs to compensate them for their forgone outside option. In equilibrium, this outside option is appropriation and deteriorates with taxation and enforcement. It is thus always beneficial for the planner to tax producers slightly more and hire more enforcement personnel. Doing so absorbs potential appropriators and induces more agents to produce. The ability to subsidize unproductive producers strengthens this effect. Third, in equilibrium, enforcement personnel is recruited from a pool of agents that would otherwise engage in appropriation, not in production. Using unproductive rather than productive factors makes perfectly secure property rights affordable.

The literature explicitly linking taxation and property rights enforcement often focuses on state capacity for these two activities (e.g., Besley and Persson (2009, 2010), Herrera and Martinelli (2013)) and on rulers or elites that use that capacity to serve their own interests (e.g., Olson (1993), Moselle and Polak (2001), Konrad and Skaperdas (2012), and Acemoglu (2005)). When considering taxation, important frictions arise from asymmetric information, which I model here along the lines of Lacker and Weinberg (1989). Grochulski (2007) and Casamatta (2011) study optimal taxation when taxes can be avoided by hiding income from taxation at a cost. While the environment here features a similar ex post moral hazard problem, it differs as agents can choose among three occupations which are directly affected or even established by the planner's choice of taxation and enforcement. The focus of this paper is also different: it is on the consequences of the informational friction for the security of property rights rather than for the optimal tax schedule. I do not address what determines the cost of hiding income from taxation. This endogenous economic institution is the subject of the literature on optimal income tax enforcement started by Reinganum and Wilde (1985), which connects to the literature on tax evasion started by Allingham and Sandmo (1972). However, while the exogenous cost of hiding income from taxation precludes a sensible welfare analysis, my results suggest that societies could benefit from increasing it.

I present the model in Section 2, its predictions in Section 3, and a discussion in Section 4.

2 The Model

There is a unit measure of risk neutral agents of two types and a benevolent planner. A measure $\mu_l > 0$ of agents has low productivity $w_l > 0$ and a measure $\mu_h > 0$ of agents has high productivity $w_h > w_l$, where $\mu_l + \mu_h = 1$ and $\bar{w} = \mu_l w_l + \mu_h w_h$ is the average productivity. Individual productivities are private information and cannot be verified by the planner. There are not "too few" highly productive agents: $\mu_h \geq 0.15$. Agents are endowed with one unit of time that is supplied indivisibly to one of three mutually exclusive occupations. An agent of type $i \in \{l, h\}$ can decide to either produce w_i units of the consumption good, work as enforcement personnel for a fixed and certain wage w_e, or engage in appropriation activities. Let χ_i^j be the share of agents of type $i \in \{l, h\}$ in occupation $j \in \{p, e, a\}$, where p, e, and a indicate production, enforcement, and appropriation, respectively.

After production, producers of type i display an income z_i to the planner and pay a tax, or collect a subsidy, $t(z_i)$ that is indexed by and may vary with the income they display.[4] A tax payment cannot exceed actual income. Producers of type h can pretend to be of type l by displaying $z_h = w_l$ so as to pay a different tax. To do so, they have to hide income $w_h - w_l$ at a cost captured by the function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$. While hiding nothing is costless, $\psi(0) = 0$, the cost of hiding an amount $x > 0$ is strictly positive, but less than x: $0 < \psi(x) < x$. Letting $\phi \equiv \psi(w_h - w_l)$, thus $\phi \in (0, (w_h - w_l))$. All income not hidden is publicly observable. Producers can neither display more income than they have nor hide all of it.[5] The income that all agents of type l could generate is at least as high as the potential hidden income of all agents of type h: $\mu_l w_l \geq \mu_h (w_h - w_l)$. I discuss this and other assumptions in Section 4.

The tax receipts are used to hire enforcers. They, and only they, can apprehend agents that appropriated other's resources, and only those. They are recognizable and safe from appropriation. Next, there are two rounds of random matching between agents. In the first round, appropriation takes place. Every agent can meet either an appropriator, a producer, or an enforcer. For any agent, the probability of meeting an agent of a certain type in a certain occupation is equal to the measure of agents of that type in that occupation. In particular, the probability p of meeting an appropriator equals the measure of appropriators, $\chi_l^a \mu_l + \chi_h^a \mu_h$, i.e., the share of appropriators in the population. Similarly, the probability $(1 - \theta)$ of meeting an enforcer is $\chi_l^e \mu_l + \chi_h^e \mu_h$; the probability of meeting a producer of type i is $\chi_i^p \mu_i$; the probability q of meeting any producer is $\chi_l^p \mu_l + \chi_h^p \mu_h$. If a producer meets an appropriator, then the appropriator takes and runs off with all her resources, irrespective of whether or not they were hidden. That is, while producers can hide income from the planner, they cannot hide it from appropriators in the same way, as the latter do not face institutional or resource

[4] Following Lacker and Weinberg (1989), this assumption is without loss of generality (see Appendix A).

[5] At the solution of the planner's problem, agents of type l do not want to display w_h, even if it is costless.

Table 1

Timing

1. Regime Choice	2. Production	3. Enforcement	4. Meetings	5. Consumption
The planner chooses and enacts a regime.	Agents choose occupations and producers produce.	Taxes are collected and enforcement personnel is paid.	Appropriation and apprehension take place.	Agents consume the resources they have in hand.

constraints, once they target somebody's income. In all other meetings, no relevant interaction takes place: appropriators do not carry resources and enforcers are recognizable, so that appropriation is not attempted.

In the second round, apprehension takes place. Successful appropriators are randomly matched with (i.e., run into) another agent. If a successful appropriator meets an enforcer, then the enforcer takes all the resources the appropriator carries and returns them to the producer who was expropriated of them. That is, the probability of apprehension equals the share of enforcement personnel in the population. The apprehended appropriator does not incur any additional costs besides zero consumption. In all other meetings, no relevant interaction takes place: successful appropriators cannot appropriate any more resources, cannot be expropriated by unsuccessful appropriators, and can be apprehended only by enforcers. After the second round, all agents consume the resources they carry.

The planner knows the economic fundamentals summarized by the productivity distribution captured by (μ_l, μ_h, w_l, w_h) and the cost parameter ϕ. At the outset, she chooses shares of agents of each type in each occupation, which imply a measure of enforcement personnel to be employed, the tax schedule, and the wage paid to enforcers in order to maximize aggregate welfare with equal weights for all agents. After production, she collects taxes from producers, and pays enforcement personnel. She has to maintain a balanced budget: the expenses for wages paid to enforcement personnel have to equal the taxes collected, minus any subsidies.

Table 1 summarizes the timing. Assumption 1 summarizes the parameter restrictions, which are sufficient but not necessary for the results. See Section 4 for a discussion.

Assumption 1. *The productivity distribution satisfies* $\mu_h \geq 0.15$ *and* $\mu_l w_l \geq \mu_h(w_h - w_l)$.

3 Analysis

I first specify the actions agents can take and the payoff functions these map into as well as the planner's objective function and problem to then state and discuss the implications of the model. All proofs can be found in Appendix C.

3.1 Payoffs and the Planner's Problem

A regime $\sigma \equiv (\chi_l^p, \chi_l^e, \chi_l^a, \chi_h^p, \chi_h^e, \chi_h^a, t_l, t_h, w_e) \in \Sigma \equiv [0,1]^6 \times [-w_h, w_h]^2 \times [0, w_h]$ collects the occupational assignments as captured by the shares of agents of each type in each occupation, the tax schedule, and the enforcement sector wage. While other (large enough) bounds would work, a natural bound on taxes, subsidies, and wages in enforcement might be the income w_h the most productive agents in the economy can generate. The planner takes into account the map from occupational choices (instructions) to the probabilities q, $(1-\theta)$, and p of meeting a producer, an enforcer, or an appropriator, respectively. She understands that

$$q = \chi_l^p \mu_l + \chi_h^p \mu_h; \quad (1-\theta) = \chi_l^e \mu_l + \chi_h^e \mu_h; \quad p = \chi_l^a \mu_l + \chi_h^a \mu_h.$$

After displaying income z_i at the cost $\psi(w_i - z_i)$ and paying taxes $t(z_i)$, a producer with productivity w_i carries resources $w_i - t(z_i) - \psi(w_i - z_i)$. Then, a producer meets an appropriator with probability p, in which case she is expropriated of all her resources. With probability $(1-\theta)$ an enforcer apprehends the appropriator and returns the resources; with probability θ, the appropriator can run off with them. Therefore, the expected payoff an agent of type i derives from production, when displaying income z_i and paying both the designated tax $t(z_i)$ for producers displaying z_i and the associated cost $\psi(w_i - z_i)$ of hiding income is given by

$$[(1-\theta)p + (1-p)]\,(w_i - t(z_i) - \psi(w_i - z_i)) = (1-\theta p)(w_i - t(z_i) - \psi(w_i - z_i)).$$

When a producer chooses an income to display, she maximizes her payoff from doing so. However, the income display must be feasible and, given that the planner understands the incentives to falsify income, consistent with all agents' occupational choices: First, producers can neither display a higher income than they have generated nor hide all income. Second, if a producer were to display an income that, given the regime, no agent that produces generates, then the planner catches that and punishes this producer prohibitively high. That is, the only income display that is feasible for producers of type l is $z_l = w_l$. The associated tax payment is $t(w_l) = t_l$. The expected payoff of a producer of type l is given by the function $\varphi_l : \Sigma \to \mathbb{R}$,

$$\varphi_l(\sigma) = (1-\theta p)(w_l - t_l). \tag{1}$$

A producer of type h can display $z_h \in \{w_l, w_h\}$, if some agents of type l produce. If no agent of type l produces, then the only income a producer of type h can display that is consistent with σ is $z_h = w_h$. The tax payment associated with the income display w_h is $t(w_h) = t_h$. Thus, given any regime $\sigma \in \Sigma$, the income display $\zeta(\sigma)$ of a producer of type h satisfies

$$\zeta(\sigma) \in \arg\max_{z \in Z(\sigma)} \; w_h - t(z) - \psi(w_h - z),$$

where

$$Z(\sigma) = \begin{cases} \{w_l, w_h\} & \text{if } \chi_l^p > 0, \\ \{w_h\} & \text{otherwise,} \end{cases}$$

is the set of all income displays z_h that are consistent with the regime σ. To simplify notation, I suppress the dependence on the regime σ and write ζ to mean $\zeta(\sigma)$ whenever there is no risk of confusion. At the optimal income display ζ, the cost of hiding resources, $\psi(w_h - \zeta)$, is either zero if $\zeta = w_h$, or ϕ if $\zeta = w_l$. The expected payoff an agent of type h can derive from producing and optimally displaying income is thus given by the function $\varphi_h : \Sigma \to \mathbb{R}$,

$$\varphi_h(\sigma) = (1-\theta p)(w_h - t(\zeta) - \psi(w_h - \zeta)) = (1-\theta p)\max\{w_h - t_h, w_h - t_l - \phi\}. \tag{2}$$

Given σ, only appropriators that meet a producer carry resources after the first round of meetings. In the second round of meetings, those successful appropriators escape apprehension with probability θ. Only in this case do appropriators carry any resources after the meetings. Therefore, the expected payoff from appropriation is proportional to the expectation of a random draw from the set of producer incomes net of taxes and costs of hiding income. It is independent of the appropriator's productivity and thus the same for all appropriators. The expected payoff from appropriation can be written as a function $\nu : \Sigma \to \mathbb{R}$ given by

$$\nu(\sigma) = \theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t(\zeta) - \psi(w_h - \zeta))\right]. \tag{3}$$

Any agent that works as enforcement personnel receives a wage w_e with certainty and her payoff from working in enforcement is thus w_e. The planner's balanced budget is given by

$$(\chi_l^e \mu_l + \chi_h^e \mu_h) w_e = \chi_l^p \mu_l t_l + \chi_h^p \mu_h t(\zeta);$$

while her objective function is aggregate welfare with equal weights for all agents:

$$\chi_l^p \mu_l \varphi_l(\sigma) + \chi_h^p \mu_h \varphi_h(\sigma) + (\chi_l^e \mu_l + \chi_h^e \mu_h) w_e + (\chi_l^a \mu_l + \chi_h^a \mu_h)\nu(\sigma).$$

Using the payoff expressions, the balanced budget constraint, and the definitions of the probabilities, which the planner understands, it can be written as (see Appendix B)

$$\chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h - \chi_h^p \mu_h \psi(w_h - \zeta). \tag{4}$$

That is, the planner's problem is to choose a regime σ consisting of shares of agents of each type in each occupation, a tax schedule, and a wage in enforcement in order to maximize total production minus the cost of hiding all income that is being hidden. As she cannot dictate individual occupations, her choice of a regime needs to be incentive compatible in the sense that no agents should expect a higher payoff from switching occupations. In addition, the nonnegative budget has to be balanced and the shares of agents of each type in each occupation have to be nonnegative and add up to one. The planner's problem is

$$\text{(PP)} \qquad \max_{\sigma \in \Sigma} \quad \chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h - \chi_h^p \mu_h \psi(w_h - \zeta)$$

$$\text{(5)} \qquad s.t. \quad (\chi_l^e \mu_l + \chi_h^e \mu_h) w_e = \chi_l^p \mu_l t_l + \chi_h^p \mu_h t(\zeta) \geq 0;$$

$$\text{(6)} \qquad \varphi_i(\sigma) \geq \max\{w_e, \nu(\sigma)\} \text{ if } \chi_i^p > 0, \ \forall i;$$

$$\text{(7)} \qquad w_e \geq \max\{\varphi_i(\sigma), \nu(\sigma)\} \text{ if } \chi_i^e > 0, \ \forall i;$$

$$\text{(8)} \qquad \nu(\sigma) \geq \max\{\varphi_i(\sigma), w_e\} \text{ if } \chi_i^a > 0, \ \forall i;$$

$$\text{(9)} \qquad \chi_i^p + \chi_i^e + \chi_i^a = 1, \ \forall i; \ \chi_i^p, \chi_i^e, \chi_i^a \geq 0, \ \forall i;$$

$$\text{(10)} \qquad q = \chi_l^p \mu_l + \chi_h^p \mu_h; \ (1 - \theta) = \chi_l^e \mu_l + \chi_h^e \mu_h; \ p = \chi_l^a \mu_l + \chi_h^a \mu_h.$$

Constraint (5) requires the budget to be balanced. Constraint (6) says that a producer, that may choose to display a false income if she is of type h, should be at least as well off as she would be by taking up an activity in enforcement or appropriation. Similarly, constraints (7) and (8) require agents of either type that work in enforcement or appropriation to find it optimal to do so, instead of deviating to, for example, producing and hiding income in case they are of type h. Constraint (9) requires that the shares of agents of each type in each occupation are nonnegative and add up to one. Constraint (10) states that the planner understands and takes into account how the shares of agents in all occupations map into the probabilities that enter the expected payoffs.

3.2 Anarchy and The First-Best Outcome

Before I study Problem (PP), in this section, I discuss two extreme scenarios: anarchy and the first-best outcome. First, suppose that there is neither a planner nor any other governing authority so that no taxes are collected and no enforcement personnel is hired, i.e., $\chi_l^e = \chi_h^e = t_l = t_h = w_e = 0$. In this case, in an equilibrium, all agents choose to engage in either production or appropriation in order to maximize their expected payoff, taking all other agents' occupational choices as given. I refer to this situation as anarchy.

Proposition 1. *In anarchy, any share of agents of type h produces, all others appropriate.*

In anarchy, appropriation implements redistribution. If some agents of type h produce output, then some redistribution takes place by appropriators taking resources from them after production. In particular, if all agents of type h produce, then appropriation redistributes income to those who are not productive enough to choose to engage in production themselves. Without taxation and enforcement, redistribution can only occur through appropriation.

I next describe the first-best outcome and show that there is a regime with incentive compatible occupations that attains it. Suppose that the planner is only constrained by technology and resources—or agents' productivities and endowments with time. That is, besides choosing a tax schedule and the enforcement sector wage, she can dictate all agents' occupations, irrespective of whether or not agents agree with those occupation assignments, and verify producers' taxable incomes. The tax schedule can depend on actual income, rather than displayed income, and no costs from hiding income are incurred. The planner only has to balance the budget and ensure that all shares are nonnegative and add up to one. Her problem is

$$\text{(FBP)} \qquad \begin{aligned} &\max_{\sigma\in\Sigma} \quad \chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h \\ &s.t. \quad (\chi_l^e \mu_l + \chi_h^e \mu_h) w_e = \chi_l^p \mu_l t_l + \chi_h^p \mu_h t_h \geq 0; \\ &\qquad\quad \chi_i^p + \chi_i^e + \chi_i^a = 1,\ \forall i;\ \chi_i^p, \chi_i^e, \chi_i^a \geq 0,\ \forall i. \end{aligned}$$

The following proposition characterizes the first-best outcome.

Proposition 2. *The first-best regime has all agents produce, collects total tax receipts equal to zero, and implements no enforcement at all.*

This first-best outcome derives from two aspects. First, both appropriation and enforcement are unproductive. Any agent that appropriates or enforces does not produce and thus not contribute to the pie the planner has available for distribution. Second, linear utility implies that the planner does not have an a priori incentive to redistribute resources among agents through transfers. Hence, the planner instructs all agents to produce so as to maximize output. Any tax schedule that at most redistributes income through tax payments and subsidies is efficient—not implementing redistribution through the tax schedule at all thus is.

If the planner cannot dictate individual occupational choices, then she chooses incentive compatible occupations, the wage paid in enforcement, and the tax schedule, which can still depend on actual income. Her problem is

$$\text{(FBP}'\text{)} \qquad \max_{\sigma\in\Sigma} \quad \chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h$$

$$s.t. \quad (\chi_l^e \mu_l + \chi_h^e \mu_h) w_e = \chi_l^p \mu_l t_l + \chi_h^p \mu_h t_h \geq 0;$$

$$(1-\theta p)(w_i - t_i) \geq \max\left\{w_e, \theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t_h)\right]\right\} \text{ if } \chi_i^p > 0,\ \forall i; \tag{11}$$

$$w_e \geq \max\left\{(1-\theta p)(w_i - t_i), \theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t_h)\right]\right\} \text{ if } \chi_i^e > 0,\ \forall i; \tag{12}$$

$$\theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t_h)\right] \geq \max\left\{(1-\theta p)(w_i - t_i), w_e\right\} \text{ if } \chi_i^a > 0,\ \forall i; \tag{13}$$

$$\chi_i^p + \chi_i^e + \chi_i^a = 1,\ \forall i;\ \chi_i^p, \chi_i^e, \chi_i^a \geq 0,\ \forall i;$$

$$q = \chi_l^p \mu_l + \chi_h^p \mu_h;\ (1-\theta) = \chi_l^e \mu_l + \chi_h^e \mu_h;\ p = \chi_l^a \mu_l + \chi_h^a \mu_h. \tag{14}$$

The additional constraints (11)–(13) ensure that all agents can expect a payoff from their occupation—production, enforcement, or appropriation—that is at least as high as the maximum payoff they could obtain from either one of the alternative occupations. Constraint (14) ensures that the occupational choices are consistent with the probabilities of meeting agents in certain occupations. The solution to Problem (FBP′) implements the first-best outcome.

Proposition 3. *There is a unique first-best regime with incentive compatible occupations. It has all agents produce, equalizes after-tax incomes, and implements no enforcement at all.*

The planner instructs all agents to produce so as to maximize the pie. She then collects taxes and subsidizes unproductive agents with transfers paid for by productive agents. The transfers equalize consumption across agents so as to incentivize abstention from appropriation in favor of production. No resources are spent on enforcement at all. The intuition is unchanged: appropriation and enforcement do not contribute to the pie available to society.

3.3 Unverifiable Incomes

In this section, I study Problem (PP). A number of insights allow to simplify the problem. First, as is intuitive, the planner chooses to employ the most productive agents in production.

Lemma 1. *All agents of type h produce.*

A regime in which all agents of type h produce, all agents of type l appropriate, and no taxes are collected at all satisfies all constraints of Problem (PP) and attains a value of $\mu_h w_h$. Suppose that a strictly positive share of agents of type l produces, and that those agents find it optimal to do so. Then, $\theta p < 1$ and the after-tax income matters. As the cost ϕ of hiding income $(w_h - w_l)$ satisfies $\phi < (w_h - w_l)$, the after-tax income of producers of type h is

$$\max\{w_h - t_h, w_h - t_l - \phi\} \geq w_h - t_l - \phi > w_h - t_l - (w_h - w_l) = w_l - t_l. \tag{15}$$

That is, the payoff an agent of type h derives from production is strictly greater than that of an agent of type l. Hence, if agents of type l at least weakly prefer production over the other two occupations, then all agents of type h strictly prefer production over the other occupations. Individually optimal occupational choice then requires that all agents of type h produce. That is, production of all agents of type h is attainable; and if agents of type l produce, then so do all agents of type h. Following Lemma 1, I conjecture that the planner also has some agents of type l produce; and I verify this guess via the rest of the analysis.

Conjecture 1. *Some agents of type l produce.*

Starting from Conjecture 1, the planner's regime choice departs from the first-best regime.

Lemma 2. *Some enforcement is implemented.*

As indicated by inequality (15), the planner cannot equalize after-tax incomes, because producers of type h can hide income from taxation. Thus, redistributive taxation alone cannot sufficiently counteract the incentive to engage in appropriation activities when other agents produce. The planner has to hire agents of type l as enforcement personnel. As becomes clear in the proof of Lemma 2, if there was no enforcement at all, then these agents would choose to be appropriators. That is, enforcement personnel is hired from a pool of agents that would otherwise engage in appropriation. When raising taxes to pay for them, in order to prevent deadweight loss, the planner avoids tax schedules that induce income to be hidden.

Proposition 4. *The tax schedule precludes income hidden from taxation.*

This result resembles the no-falsification theorem proven by Grochulski (2007). For any regime that induces producers of type h to hide income from taxation, there is an alternative regime in the constraint set, with a different tax schedule, that induces producers to display their true income and increases welfare. The resources that would be spent on hiding income in the original regime are collected as additional tax receipts that can be used to lower taxes on producers and pay higher wages to enforcers. As the shares of agents of each type in each occupation are the same in both regimes, they produce the same output. However, unlike in the original regime, in the alternative regime all output is consumed.

From Proposition 4 it therefore follows that income displays are truthful so that $\zeta = w_h$, $\psi(w_h - \zeta) = 0$, and $t(\zeta) = t_h$. As all agents of type h produce by Lemma 1, which implies that $\theta p < 1$, it then follows from equation (2) that the tax schedule has to satisfy

$$w_h - t_h \geq w_h - t_l - \phi. \tag{16}$$

Agents of type h should not be willing to display income w_l and incur both the associated tax payment t_l and the cost ϕ of hiding output $(w_h - w_l)$. This requirement can be rewritten as

$$t_h \leq t_l + \phi. \tag{17}$$

Inequality (17) reveals that the tax schedule cannot be too steep: the tax payment t_h designated for a producer with high income w_h cannot be too much higher than the one designated for a producer with lower income w_l. As one would expect, the upper bound on the difference between the two designated tax payments is determined by the cost of hiding income from taxation. Together, and as taxes cannot exceed incomes, inequalities (15) and (16) imply

$$w_h - t_h > w_l - t_l \geq 0. \tag{18}$$

That is, more productive producers require a higher after-tax income than less productive producers. As $\chi_l^p > 0$, it then follows from constraint (6) that agents of type h strictly prefer production over the other two occupations, which is consistent with $\chi_h^p = 1$ and $\chi_h^e = \chi_h^a = 0$. The constraints (6), (7), and (8) corresponding to agents of type h are thus not needed. For agents of type l, constraints (6), (7), and (8) imply indifference between all occupations.

Lemma 3. *Agents of type l are indifferent among occupations.*

As some agents of type l produce and some enforce, constraints (6) and (7) have to hold for $i = l$. Combining them, agents of type l should derive the same expected payoff from production and enforcement. If the share of agents of type l in appropriation is strictly positive, then appropriation should provide the same expected payoff as well. However, the expected payoffs to agents of type l from all three occupations have to be equal to each other even when no agent of type l engages in appropriation activities. The reason is that, in order for them to abstain from appropriation in favor of another activity, agents of type l need only be made at least as well off as if they engaged in appropriation. If production and enforcement promised a strictly greater expected payoff than appropriation, then the planner could afford to move some enforcement personnel into the production sector, thereby increasing output, adjusting taxes appropriately, without changing the relevant meeting probabilities enough to make appropriation profitable. That is, a regime that offers agents of type l a strictly higher expected payoff in production and enforcement than it offers in appropriation is dominated by another regime in the constraint set that does not.

Combining the insights of Lemmas 1–3 and Proposition 4, together with Conjecture 1, Problem (PP) can be simplified: maximizing the objective function (4) amounts to maximizing χ_l^p, focusing on regimes in the subset $\Sigma' \equiv [0,1]^3 \times \{1\} \times \{0\} \times \{0\} \times [-w_h, w_h]^2 \times [0, w_h] \subset \Sigma$ with generic element $\sigma = (\chi_l^p, \chi_l^e, \chi_l^a, 1, 0, 0, t_l, t_h, w_e)$. The simplified planner's problem is

$$\text{(PP}'\text{)} \qquad \max_{\sigma \in \Sigma'} \quad \chi_l^p$$

$$\text{(19)} \qquad s.t. \quad \chi_l^e \mu_l w_e = \chi_l^p \mu_l t_l + \mu_h t_h \geq 0;$$

$$\text{(20)} \qquad t_l + \phi \geq t_h;$$

$$\text{(21)} \qquad \varphi_l(\sigma) = w_e;$$

$$\text{(22)} \qquad w_e = \nu(\sigma);$$

$$\text{(23)} \qquad \chi_l^p + \chi_l^e + \chi_l^a = 1; \; \chi_l^p, \chi_l^e, \chi_l^a \geq 0;$$

$$\text{(24)} \qquad q = \chi_l^p \mu_l + \mu_h; \; (1-\theta) = \chi_l^e \mu_l; \; p = \chi_l^a \mu_l.$$

Problem (PP′) has a solution because the objective function as well as all functions involved in the constraints are continuous and Σ' is compact. If that solution verifies Conjecture 1, then it solves Problem (PP) as, by Lemmas 1–3 and Proposition 4—or, more precisely, by the specifics of their proofs detailed in Appendix C—all regimes in its constraint set are dominated by some regime in the constraint set of Problem (PP′).

The planner's objective function is simply aggregate production, which is maximized by maximizing the share of agents of type l that join the agents of type h in production. Constraints (21) and (22) together state that an agent of type l should expect the same payoff from all three occupations. Both appropriators and enforcers are agents of type l and cannot pretend to have produced w_h were they to deviate to production. As before, but accounting for the fact that all agents of type h produce, constraint (19) is the budget balance constraint; constraint (23) requires that the shares of agents of type l in all occupations are nonnegative and add up to one; and the equalities collected in constraint (24) state that the planner understands and takes into account how these shares map into the meeting probabilities. Constraint (20) indicates that the difference between the tax payments designated for producers of types l and h is bounded by the cost of hiding income from taxation. In fact, the planner exhausts this bound by setting taxes that make producers of type h indifferent among income displays.

Lemma 4. *Producers of type h are indifferent among income displays.*

The planner implements a tax schedule such that $t_h = t_l + \phi$. Redistributive taxation is one means of inducing rather unproductive agents to abstain from appropriation and engage in production instead. Unlike enforcement personnel, who are paid from the tax receipts collected, agents of type l that redistribution induces to produce do contribute to the pie available to society. The planner's regime exhausts the bound on the difference between the taxes facing productive and unproductive producers to exploit the advantages of redistributive taxation as much as possible. This aspect plays an important role in the planner's solution.

Proposition 5. *The planner's problem has a unique solution σ^*. All agents of type h and some agents of type l produce. All agents that do not produce are employed in enforcement. If the costs of hiding income from taxation are high enough, then producers of type l receive a subsidy. More specifically, the solution $\sigma^* = (\chi_l^{p*}, \chi_l^{e*}, \chi_l^{a*}, \chi_h^{p*}, \chi_h^{e*}, \chi_h^{a*}, t_l^*, t_h^*, w_e^*)$ sets*

$$\chi_l^{p*} = \mu_l^{-1}(\theta^* - \mu_h) > 0, \quad \chi_l^{e*} = 1 - \chi_l^{p*} > 0, \quad \chi_l^{a*} = 0, \tag{25}$$

$$\chi_h^{p*} = 1, \quad \chi_h^{e*} = 0, \quad \chi_h^{a*} = 0, \tag{26}$$

$$w_e^* = \frac{\theta^*}{1 + (1 - \theta^*)\theta^*} \left(\chi_l^{p*} \mu_l w_l + \mu_h w_h\right), \quad t_l^* = w_l - w_e^*, \quad t_h^* = t_l^* + \phi, \tag{27}$$

where θ^* uniquely solves

$$\frac{\theta}{1 - \theta^2} \mu_h (w_h - w_l - \phi) - \theta w_l - \mu_h \phi = 0. \tag{28}$$

That is, Proposition 5 verifies Conjecture 1: some agents of type l produce. The planner chooses to employ enough enforcement personnel so as to crowd out appropriation altogether, and property rights are perfectly secure. She does so by absorbing potential appropriators into the sector for enforcement for a certain wage. Appropriation is never attempted. Enforcement personnel does not actually provide a service other than being present—and thus effectively deterring appropriation efforts. Being employed in enforcement largely amounts to refraining from appropriation. The wage paid in enforcement amounts to a transfer financed by taxing agents that produce. Employment in enforcement thus institutionalizes redistribution that would otherwise take place through appropriation.

While taxes may distort the occupational choice between production and appropriation, higher tax payments decrease the expected payoffs from both activities. This effect is present in general, independent of the exact tax schedule in place, and as long as utility is increasing. Thus, taxation affects the choice between either one of those two occupations and employment in enforcement. Too high taxes may draw productive agents (as well as appropriators) into the enforcement sector. The threat of expropriation of the resources an agent carries, however, affects the occupational choice between production and appropriation—as does the probability of apprehension. Intermediated by the sector for enforcement, taxation increases the probability of productive agents being able to reap the returns to their productive activity, which increases the incentive to produce. At the same time, the incentive to engage in appropriation activities decreases because the probability of escaping apprehension after successful appropriation decreases. That is, more agents prefer to produce. This effect, too, is present in general, independent of the exact tax schedule in place, and as long as utility is increasing. In addition, unproductive agents lean more towards switching to an occupation in the enforcement sector. Thus, the planner optimally employs rather unproductive members of society that do not produce as enforcement personnel at a wage that makes them indifferent between enforcement and appropriation. As she absorbs all potential appropriators into the enforcement sector, there is no appropriation and property rights are perfectly secure.

This security of property rights derives from the availability of a sector for enforcement of those rights because it provides an alternative occupation. That occupation is unproductive, as is appropriation. However, in contrast to appropriation, it does not harm the incentives of more productive agents to actually produce. Perfectly secure property rights are not too costly because enforcement personnel is recruited from a pool of agents that would otherwise engage in appropriation. While enforcers do not produce, enforcement does not withdraw agents from productive activities. Moreover, the resources spent on enforcement finance the consumption of enforcement personnel, which the planner values.

The tax profile that finances enforcement is redistributive. High income agents pay higher taxes than—and may even finance transfers to—low income agents. In fact, the tax facing producers of type l is negative if the cost of hiding income is high enough. Negative taxes for unproductive producers subsidize their production, which contributes to the pie society has available for distribution amongst its members. These subsidies also incentivize abstention from appropriation, which would hurt others' incentives to produce. Redistributive taxation thus plays an important role, and it is easier to implement it in societies in which it is more costly for agents to hide income from taxation.

Proposition 6. *Societies with higher costs of hiding income from taxation enact a steeper tax profile with a smaller minimum tax, employ fewer enforcement personnel at a higher wage, see more productive activity, produce more output, and experience higher welfare.*

That is, if the costs of hiding income from taxation are higher, then the planner can implement more redistribution through taxation. She can and does then choose to employ fewer agents in enforcement and enact a steeper tax profile. That steeper tax profile—possibly offering (larger) subsidies to unproductive producers—encourages some agents to produce that would choose to enforce if the tax schedule were to implement less redistribution. As a consequence, fewer agents work in enforcement and more agents produce more output that can be shared among the members of society, which increases welfare. As aggregate consumption rises, agents employed in enforcement share in that rise via higher wages.

4 Discussion

In this section, I briefly discuss a number of assumptions. For example, I assume that the planner maximizes aggregate welfare with equal weights for all agents. I take the view that there is no a priori reason to exclude appropriators' welfare from the planner's considerations—or assign a smaller weight to it. Appropriation is a form of redistribution; and the planner's solution institutionalizes another one, using taxation and enforcement. The assumption of linear utility ensures that the planner does not have an inherent desire to redistribute income. All redistribution the planner implements is thus driven by the need to provide agents with incentives. Strictly concave utility would intensify the planner's desire to both redistribute incomes across agents and reduce the uncertainty implied by insecure property rights.

The parameter restrictions collected in Assumption 1 are sufficient but not necessary for the results. They affect a function arising from the model, ensuring its monotonicity and restricting its behavior at the boundary of its domain. A somewhat smaller minimum share of agents of type h in the economy than 0.15 could in fact be accommodated without any other changes. More generally, these restrictions on the productivity distribution can be relaxed quite a bit, if one is willing to impose restrictions on the cost of hiding income from taxation. If one is willing to think of the U.S. as a benchmark represented by the first-best outcome in the rather stylized environment here, then Assumption 1 in principle allows the parameters to qualitatively capture at least some aspects of the U.S. income distribution. For example, according to some sources, in the U.S., between 1986 and 2013, the income share held by the top 20 percent has always been less than 50 percent, with an average of 45.6 percent.[6] That is, the income share of the lower 80 percent was larger than that of the top 20 percent. Letting $\mu_h = 0.2 > 0.15$ and $\mu_l = 0.8$, the income share of the top 20 percent in the first-best is $\frac{\mu_h w_h}{\mu_l w_l + \mu_h w_h}$, while that of the lower 80 percent is $\frac{\mu_l w_l}{\mu_l w_l + \mu_h w_h}$. Any $w_l > 0$ and $w_h > w_l$ such that $\mu_l w_l \geq \mu_h w_h$ would be qualitatively consistent with this data and satisfy the parameter restriction $\mu_l w_l \geq \mu_h(w_h - w_l)$. Other estimates of the income share held by the top 20 percent can be captured as well; and so can the income shares held by many other quantiles.

I assume that the probability of any agent meeting another agent in a certain occupation equals the measure of agents in that occupation. In particular, for any agent, the probability of meeting an appropriator equals the measure of appropriators, i.e., the share of appropriators in the population. Acemoglu (1995) and İmrohoroğlu et al. (2000), among others, use a similar specification. While it simplifies the analysis, it shares qualitative features with other specifications, such as a constant returns to scale matching function: all else equal, more appropriators and fewer producers increase the probability of meeting an appropriator. In fact, this specification is unfriendly towards the results. The assumption that the probability of apprehension equals the share of enforcement personnel in the population makes secure property rights more costly than others. Among all concave (production) functions mapping the share of enforcement personnel in the population into a probability of apprehension, with zero and one mapping into zero and one, respectively, a linear function gives a lower probability than any strictly concave function everywhere in the interior of the domain. Similarly, successful appropriators that are apprehended by enforcement personnel are not subject to any punishment other than zero consumption (see, e.g., Becker (1968), Stigler (1970), Ehrlich (1973), and many others). Here, imposing a potentially costly punishment might make it easier to induce abstention from appropriation. However, the regime the planner chooses in the present environment remains attainable in that case.

One could assume that enforcement personnel may explicitly divert the resources recovered from apprehended appropriators without changing the results. The solution to the planner's problem I analyze would still be attainable, dominate all other regimes, and leave no room for such a deviation as there is no appropriation in the first place. In fact, the possibility of corrupt enforcement personnel grabbing resources themselves is captured to the extend that appropriation activities are a stylized description of unproductive redistribution of resources, including corruption. Moreover, in order to focus on a planner facing the friction that income can be hidden from taxation, I do not consider corruptible tax collectors (see, e.g., Chander and Wilde (1992), Besley and McLaren (1993), and Hindriks et al. (1999)). I thus abstract

[6]Data from The World Bank, downloadable at http://data.worldbank.org/indicator/SI.DST.05TH.20.

from an analysis of government agents' incentives and their implications altogether (see, e.g., Becker and Stigler (1974), Basu et al. (1992), Mookherjee and Png (1995), Acemoglu and Verdier (1998, 2000), and Polinsky and Shavell (2001)). The lack of a choice of effort to exert when an agent is employed in enforcement seems to be less relevant than the possibility of diverting recovered resources. Given the latter option, enforcement personnel would want to ensure a high probability of apprehending successful appropriators.

According to the U.S. Internal Revenue Service (2016), improved visibility in the sense of information reporting and tax withholding improves tax compliance. Andreoni et al. (1998) argue that it is more difficult to hide income from taxation in developed countries than it is to do the same in developing ones, and that income derived from farms or sole proprietorship is particularly prone to evasion (p. 821). In addition, Schneider and Enste (2000) point to a role for the informal sector in tax evasion and provide estimates of its size to show that it is more prevalent in developing and transition economies than in developed countries. Therefore, a possible interpretation of the level of costs of hiding income from taxation might be as follows. One could imagine a society with a well governed and equipped tax authority; with a well developed financial market in which participants have effective screening devices available, and many companies are required to regularly report to, and are subject to audits by agents of, other participants; with a relatively important heavy and manufacturing industry, and a relatively less important agrarian sector; and with relatively many big enough companies that require a well established organizational form with cross-checks and within-firm bureaucracy to operate effectively. One could also imagine another society with an understaffed tax authority; with a large informal sector; with a relatively important agrarian sector; with large rural areas that are far behind the urban centers, both technologically and administratively; in which most firms are small enterprises, with a single owner often being the single employee. One might expect that it would be more difficult and thus costly to hide income and escape reporting duties in the former society.

5 Conclusion

In a model of appropriation and endogenous enforcement of property rights, I analyzed what a planner can achieve when she is unable to verify taxable incomes. I find that this friction cannot by itself help explain imperfectly secure property rights. Although it induces a binding and effective constraint on the planner's regime choice, it does not prevent her from implementing perfectly secure property rights. To do so, she uses a mix of redistributive taxation and employment of potential appropriators in enforcement. When higher costs of hiding income from taxation allow for more redistribution through taxation, the planner chooses to absorb fewer agents in enforcement and subsidize more unproductive producers.

Appendices

A Tax Schedules

I briefly replicate the argument given in Lacker and Weinberg (1989) for this environment. A mechanism consists of a message space M and a tax schedule t that maps the message and the income displayed into $\mathbb{R}$. A producer w chooses a message $m(w) \in M$ and an income display $z(w)$ to maximize $w - t(m(w), z(w)) - \psi(w - z(w))$. Suppose two agents with different productivities w_1 and $w_2 \neq w_1$ were to send different messages $m(w_1) = m_1$ and $m(w_2) = m_2 \neq m_1$ but display the same income $z(w_1) = z(w_2) = \hat{z}$. By optimality of message and display, for agent w_1, $t(m_1, \hat{z}) \leq t(m_2, \hat{z})$, while for agent w_2, $t(m_1, \hat{z}) \geq t(m_2, \hat{z})$, so that $t(m_1, \hat{z}) = t(m_2, \hat{z})$. The same income display implies the same tax payment, irrespective of the message, which justifies focusing on tax schedules that only depend on displayed income.

B The Planner's Objective Function

Using the payoff expressions, the balanced budget constraint, and the definitions of the probabilities, which the planner understands, the planner's objective function is given by

$$\begin{aligned}
&\chi_l^p \mu_l \varphi_l(\sigma) + \chi_h^p \mu_h \varphi_h(\sigma) + (\chi_l^e \mu_l + \chi_h^e \mu_h) w_e + (\chi_l^a \mu_l + \chi_h^a \mu_h) \nu(\sigma) \\
&= (1 - \theta p)[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t(\zeta))] - (1 - \theta p) \chi_h^p \mu_h \psi(w_h - \zeta) \\
&\quad + [\chi_l^p \mu_l t_l + \chi_h^p \mu_h t(\zeta)] + \theta p\, [\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t(\zeta))] - \theta p \chi_h^p \mu_h \psi(w_h - \zeta) \\
&= \chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t(\zeta)) - \chi_h^p \mu_h \psi(w_h - \zeta) + \chi_l^p \mu_l t_l + \chi_h^p \mu_h t(\zeta) \\
&= \chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h - \chi_h^p \mu_h \psi(w_h - \zeta).
\end{aligned}$$

C Proofs

Proposition 1

Proof. In anarchy, $\sigma = (\chi_l^p, 0, \chi_l^a, \chi_h^p, 0, \chi_h^a, 0, 0, 0)$, $\varphi_i(\sigma) = (1-p)w_i$, $i = l, h$, and $\nu(\sigma) = (\chi_l^p \mu_l w_l + \chi_h^p \mu_h w_h)$, where $p = \chi_l^a \mu_l + \chi_h^a \mu_h$ and $\chi_i^a = 1 - \chi_i^p$, $i = l, h$. First, in equilibrium, $\chi_l^p = 0$. Suppose for a contradiction that $\chi_l^p > 0$. Then, it must hold that $\varphi_h(\sigma) > \varphi_l(\sigma) \geq \nu(\sigma)$ so that all agents of type h produce, $\chi_h^p = 1$. It follows that $\nu(\sigma) = (\chi_l^p \mu_l w_l + \mu_h w_h) > (\chi_l^p \mu_l + \mu_h) w_l = (1-p) w_l = \varphi_l(\sigma)$, a contradiction. Thus, $\chi_l^p = 0$ and $\chi_l^a = 1$. For any $\chi_h^p \in [0,1]$, $\varphi_h(\sigma) = (1-p)w_h = (1 - \mu_l - \chi_h^a \mu_h) w_h = (\mu_h - \chi_h^a \mu_h) w_h = (1 - \chi_h^a) \mu_h w_h = \chi_h^p \mu_h w_h = \nu(\sigma) > \chi_h^p \mu_h w_l = (1-p) w_l$. That is, any profile of occupational choices that has any share of agents of type h producing and all other agents appropriating maximizes all agents' expected payoffs, given all others' occupational choices, and thus is an equilibrium. ∎

Proposition 2

Proof. Consider Problem (FBP). The objective function is always less than or equal to $\bar{w}$ and equals $\bar{w}$ if and only if $\chi_l^p = \chi_h^p = 1$ so that $\chi_l^e = \chi_h^e = 0$, implying $\chi_l^p \mu_l t_l + \chi_h^p \mu_h t_h = 0$. ■

Proposition 3

Proof. Consider Problem (FBP′). The objective function is always less than or equal to $\bar{w}$ and equals $\bar{w}$ if and only if $\chi_l^p = \chi_h^p = 1$ so that $\chi_l^e = \chi_h^e = 0$, implying $\chi_l^p \mu_l t_l + \chi_h^p \mu_h t_l = 0$. Consider the regime given by $\chi_l^p = \chi_h^p = 1$, $\chi_l^e = \chi_h^e = \chi_l^a = \chi_h^a = 0$, $t_l = w_l - \bar{w}$, $t_h = w_h - \bar{w}$, and $w_e = 0$. The tax receipts equal $\mu_l(w_l - \bar{w}) + \mu_h(w_h - \bar{w}) = \mu_l w_l + \mu_h w_h - (\mu_l + \mu_h)\bar{w} = \bar{w} - \bar{w} = 0$. As $p = 0$, $(1 - \theta p)(w_i - t_i) = \bar{w}$; as $\theta = 1$, $\theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t_h)\right] = \bar{w}$. Thus, this regime satisfies all constraints of Problem (FBP′) and implements the first-best. It is the only regime that does so: if $w_l - t_l \neq w_h - t_h$, then constraint (11) is violated for one of the two types of agents as $\theta\left[\chi_l^p \mu_l (w_l - t_l) + \chi_h^p \mu_h (w_h - t_h)\right] = \mu_l(w_l - t_l) + \mu_h(w_h - t_h) > \min\{w_l - t_l, w_h - t_h\}$; if $w_l - t_l = w_h - t_h$, then the balanced budget constraint, together with zero tax receipts after transfers, implies that $w_l - t_l = w_h - t_h = \bar{w}$. ■

Lemma 1

Proof. First, production by all agents of type h can be attained. Consider the regime σ that has all agents of type h produce and is given by $\chi_h^p = \chi_l^a = 1$, $\chi_h^e = \chi_h^a = \chi_l^p = \chi_l^e = 0$, $t_l = t_h = 0$, $w_e = 0$. Then, $\theta = 1$ and $p = \mu_l$ so that $\varphi_h(\sigma) = (1 - \mu_l)w_h = \mu_h w_h$, $\varphi_l(\sigma) = (1 - \mu_l)w_l = \mu_h w_l$, and $\nu(\sigma) = \mu_h w_h$. Therefore, $\varphi_h(\sigma) \geq \max\{w_e, \nu(\sigma)\}$ and $\nu(\sigma) \geq \max\{\varphi_l(\sigma), w_e\}$. Thus, σ satisfies all constraints of Problem (PP) and is thus attainable.
Second, if any agents of type l produce, then all agents of type h produce. Consider any regime σ such that $\chi_l^p > 0$ and $\chi_h^p < 1$. Then, $\theta p < 1$ so that $\varphi_h(\sigma) = (1 - \theta p)\max\{w_h - t_h, w_h - t_l - \phi\} \geq (1 - \theta p)(w_h - t_l - \phi) > (1 - \theta p)(w_h - t_l - (w_h - w_l)) = (1 - \theta p)(w_l - t_l) = \varphi_l(\sigma)$, because $w_h - w_l > \phi$. But, as $\chi_l^p > 0$, $\varphi_l(\sigma) \geq \max\{w_e, \nu(\sigma)\}$. That is, $\varphi_h(\sigma) > \max\{w_e, \nu(\sigma)\}$, which violates either constraint (7), constraint (8), or both, as either $\chi_h^e > 0$, $\chi_h^a > 0$, or both, i.e., the regime σ is not in the constraint set of Problem (PP). ■

Lemma 2

Proof. Consider any regime σ such that $\chi_l^p > 0$ and $\chi_l^e = 0$. As $\chi_h^p = 1$ by Lemma 1, $\chi_h^e = \chi_l^e = 0$ and thus $\theta = 1$. Therefore, $p = 1 - q$ so that $\varphi_l(\sigma) = (1 - p)(w_l - t_l) = q(w_l - t_l)$. At the same time, $\nu(\sigma) = \chi_l^p \mu_l (w_l - t_l) + \mu_h \max\{w_h - t_h, w_h - t_l - \phi\} \geq \chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi) > \chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - (w_h - w_l)) = \chi_l^p \mu_l (w_l - t_l) + \mu_h (w_l - t_l)) = q(w_l - t_l)$, because $w_h - w_l > \phi$. That is, $\nu(\sigma) > \varphi_l(\sigma)$, which violates constraint (6), because $\chi_l^p > 0$, i.e., the regime σ is not in the constraint set of Problem (PP). ■

Proposition 4

Proof. Consider any regime σ with $\chi_h^p = 1$, $\chi_l^p > 0$, and, by Lemma 2, $\chi_l^e > 0$ that satisfies all constraints of Problem (PP) and induces producers of type h to hide income from taxation. That is, $w_h - t_l - \phi > w_h - t_h$. The value of the objective function is $\chi_l^p \mu_l w_l + \mu_h w_h - \mu_h \phi$. I show that there is an alternative regime with unchanged occupation assignments but a different tax schedule that prevents income from being hidden and increases welfare. There are two cases: $\chi_l^a = 0$ and $\chi_l^a > 0$.

First, suppose $\chi_l^a = 0$. Then, $p = 0$, $(1-\theta) = \chi_l^e \mu_l = 1 - q$, and $\varphi_l(\sigma) = w_e \geq \nu(\sigma)$ or

$$w_l - t_l = w_e = (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l t_l + \mu_h t_l) \geq (1 - \chi_l^e \mu_l) [\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)].$$

Let $\epsilon = \frac{\mu_h}{\mu_l + \mu_h} \phi$ and let $\hat{\sigma}$ be given by $\hat{\chi}_i^j = \chi_i^j$ for all i, j, $\hat{t}_h = t_l + \phi - \epsilon$, $\hat{t}_l = t_l - \epsilon$, and $\hat{w}_e = w_e + \epsilon$. Then, producers of type h do not hide income as $w_h - \hat{t}_h = w_h - t_l - \phi + \epsilon = w_h - \hat{t}_l - \phi$, which is also greater than $w_l - \hat{t}_l$, because $w_h - w_l > \phi$, so that $\varphi_h(\hat{\sigma}) > \varphi_l(\hat{\sigma})$. Also, $\varphi_l(\hat{\sigma}) = w_l - \hat{t}_l = w_l - t_l + \epsilon = w_e + \epsilon = \hat{w}_e$ and the tax receipts increase to

$$\chi_l^p \mu_l \hat{t}_l + \mu_h \hat{t}_h = \chi_l^p \mu_l (t_l - \epsilon) + \mu_h (t_l + \phi - \epsilon) = \chi_l^p \mu_l t_l + \mu_h t_l + \mu_h \phi - q\epsilon = \chi_l^e \mu_l w_e + (1-q)\epsilon$$

or $\chi_l^e \mu_l w_e + \chi_l^e \mu_l \epsilon$, which is exactly enough to pay all $\chi_l^e \mu_l$ enforcers $\hat{w}_e$, while

$$\begin{aligned}\nu(\hat{\sigma}) &= (1 - \chi_l^e \mu_l) [\chi_l^p \mu_l (w_l - t_l + \epsilon) + \mu_h (w_h - t_l - \phi + \epsilon)] \\ &= (1 - \chi_l^e \mu_l) [\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)] + (1 - \chi_l^e \mu_l) q\epsilon < \nu(\sigma) + \epsilon.\end{aligned}$$

That is, $\varphi_l(\hat{\sigma}) = \hat{w}_e > \nu(\hat{\sigma})$ so that $\hat{\sigma}$ satisfies all constraints in Problem (PP) but yields a higher objective function value, $\chi_l^p \mu_l w_l + \mu_h w_h$, than σ, a contradiction.

Second, suppose $\chi_l^a > 0$. Then, $p > 0$ and, as σ obeys all constraints, $\varphi_l(\sigma) = w_e = \nu(\sigma)$ or

$$(1 - \theta p)(w_l - t_l) = w_e = (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l t_l + \mu_h t_l) = \theta [\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)].$$

Let $\epsilon_l = \frac{\mu_h}{\mu_l + \mu_h} \phi (1 + (1-\theta)\theta)^{-1} (\theta^{-1} - p)^{-1} > 0$ and $\epsilon_h = \frac{\mu_l + \mu_h}{\mu_h} (\theta^{-1} - p - \chi_l^p \mu_l) \epsilon_l$ and let $\hat{\sigma}$ be given by $\hat{\chi}_i^j = \chi_i^j$ for all i, j, $\hat{t}_h = t_l + \phi - \epsilon_h$, $\hat{t}_l = t_l - \epsilon_l$, and $\hat{w}_e = w_e + (1 - \theta p)\epsilon_l$. Then, $\epsilon_h > \epsilon_l$ because $\mu_h^{-1} (\theta^{-1} - p - \chi_l^p \mu_l) > 1$ as $\theta^{-1} > 1 > \chi_l^a \mu_l + \chi_l^p \mu_l + \mu_h$, so that producers of type h do not hide income as $w_h - \hat{t}_h = w_h - t_l - \phi + \epsilon_h > w_h - t_l - \phi + \epsilon_l = w_h - \hat{t}_l - \phi$, which is also greater than $w_l - \hat{t}_l$, because $w_h - w_l > \phi$, so that $\varphi_h(\hat{\sigma}) > \varphi_l(\hat{\sigma})$. Also, $\varphi_l(\hat{\sigma}) = (1 - \theta p)(w_l - \hat{t}_l) = (1 - \theta p)(w_l - t_l) + (1 - \theta p)\epsilon_l = w_e + (1 - \theta p)\epsilon_l = \hat{w}_e$ and the tax receipts increase to, using the definition of ϵ_l to replace $\mu_h \phi = (\mu_l + \mu_h)(1 + (1-\theta)\theta)(\theta^{-1} - p)\epsilon_l$,

$$\begin{aligned}\chi_l^p \mu_l \hat{t}_l + \mu_h \hat{t}_h &= \chi_l^p \mu_l (t_l - \epsilon_l) + \mu_h (t_l + \phi - \epsilon_h) \\ &= \chi_l^p \mu_l t_l + \mu_h t_l + \mu_h \phi - \mu_h \epsilon_h - \chi_l^p \mu_l \epsilon_l\end{aligned}$$

$$
\begin{aligned}
&= \chi_l^p \mu_l t_l + \mu_h t_l + (1 + (1-\theta)\theta)(\theta^{-1} - p)\epsilon_l - (\theta^{-1} - p - \chi_l^p \mu_l)\epsilon_l - \chi_l^p \mu_l \epsilon_l \\
&= \chi_l^e \mu_l w_e + (1-\theta)\epsilon_l - (1-\theta)\theta p \epsilon_l + \chi_l^p \mu_l \epsilon_l - \chi_l^p \mu_l \epsilon_l \\
&= \chi_l^e \mu_l w_e + \chi_l^e \mu_l (1 - \theta p)\epsilon_l,
\end{aligned}
$$

which is exactly enough to pay all $\chi_l^e \mu_l$ enforcers $\hat{w}_e$, while

$$
\begin{aligned}
\nu(\hat{\sigma}) &= \theta \left[\chi_l^p \mu_l (w_l - t_l + \epsilon_l) + \mu_h (w_h - t_l - \phi + \epsilon_h)\right] \\
&= \theta \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)\right] + \theta(\chi_l^p \mu_l \epsilon_l + \mu_h \epsilon_h) \\
&= \nu(\sigma) + \theta(\chi_l^p \mu_l \epsilon_l + (\theta^{-1} - p - \chi_l^p \mu_l)\epsilon_l) \\
&= \nu(\sigma) + (1 - \theta p)\epsilon_l.
\end{aligned}
$$

That is, $\varphi_l(\hat{\sigma}) = \hat{w}_e = \nu(\hat{\sigma})$ so that $\hat{\sigma}$ satisfies all constraints in Problem (PP) but yields a higher objective function value, $\chi_l^p \mu_l w_l + \mu_h w_h$, than σ, which establishes a contradiction and thereby completes the proof. ■

Lemma 3

Proof. Consider any regime σ with $\chi_h^p = 1$, $\chi_l^p > 0$, and, by Lemma 2, $\chi_l^e > 0$ that satisfies all constraints of Problem (PP). Following Proposition 4, σ also induces $\zeta = w_h$, $\psi(w_h - \zeta) = 0$, and $t(\zeta) = t_h$. First, as $\chi_l^p > 0$ and $\chi_l^e > 0$, combining constraints (6) and (7) gives

$$
\varphi_l(\sigma) \geq \max\{w_e, \nu(\sigma)\} \geq w_e \geq \max\{\varphi_l(\sigma), \nu(\sigma)\} \geq \varphi_l(\sigma),
$$

implying that $\varphi_l(\sigma) = w_e$. Second, if $\chi_l^a > 0$, then the inequality in constraint (8) has to be satisfied and combining it with constraint (7) yields

$$
\nu(\sigma) \geq \max\{\varphi_l(\sigma), w_e\} = w_e \geq \max\{\varphi_l(\sigma), \nu(\sigma)\} \geq \nu(\sigma),
$$

implying that $\varphi_l(\sigma) = w_e = \nu(\sigma)$. Finally, suppose that $\chi_l^a = 0$. Then, the inequality in constraint (8) may or may not be satisfied. Suppose that $w_e > \nu(\sigma)$. I show that there is another regime $\hat{\sigma}$ with $\hat{w}_e = \nu(\hat{\sigma})$ that is associated with higher welfare and thus dominates σ. Given the regime σ, $\chi_l^e = 1 - \chi_l^p > 0$, $p = 0$, and $\varphi_l(\sigma) = w_e > \nu(\sigma)$ or

$$
w_l - t_l = w_e = (\chi_l^e \mu_l)^{-1}(\chi_l^p \mu_l t_l + \mu_h t_h) > (1 - \chi_l^e \mu_l)\left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_h)\right] > 0.
$$

Notice that the tax receipts implied by σ are bounded away from zero by $\nu(\sigma)$ as, by inequality (18), $w_h - t_h > w_l - t_l \geq 0$. There exists an $\epsilon > 0$ such that, using $q = (\chi_l^p \mu_l + \mu_h)$,

$$
w_l - t_l + \epsilon > (\chi_l^e \mu_l)^{-1}(\chi_l^p \mu_l t_l + \mu_h t_h - q\epsilon) > (1 - \chi_l^e \mu_l)\left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_h) + q\epsilon\right].
$$

(The tax receipts are still bounded away from zero.) There exists a $\delta > 0$, $\delta < \chi_l^e$ such that

$$\begin{aligned} w_l - t_l + \epsilon &> \left(\chi_l^e \mu_l - \delta\mu_l\right)^{-1} \left(\chi_l^p \mu_l t_l + \mu_h t_h - q\epsilon + \delta\mu_l(t_l - \epsilon)\right) \\ &> \left(1 - \chi_l^e \mu_l + \delta\mu_l\right) \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h(w_h - t_h) + q\epsilon + \delta\mu_l(w_l - t_l + \epsilon)\right]. \end{aligned}$$

(The tax receipts are still bounded away from zero.) Then, there exists a $\kappa > 0$ such that

$$\begin{aligned} & w_l - t_l + \epsilon > \left(\chi_l^e \mu_l - \delta\mu_l\right)^{-1} \left(\chi_l^p \mu_l t_l + \mu_h t_h - q\epsilon + \delta\mu_l(t_l - \epsilon) - \mu_h\kappa\right) \\ & = \left(1 - \chi_l^e \mu_l + \delta\mu_l\right) \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h(w_h - t_h) + q\epsilon + \delta\mu_l(w_l - t_l + \epsilon) + \mu_h\kappa\right]. \end{aligned}$$

(The receipts are still positive.) Then, there are $\gamma_l > 0$ and $\gamma_h = -\frac{(\chi_l^p + \delta)\mu_l}{\mu_h}\gamma_l < 0$ such that

$$\begin{aligned} & w_l - t_l + \epsilon - \gamma_l \\ & = \left(\chi_l^e \mu_l - \delta\mu_l\right)^{-1} \left(\chi_l^p \mu_l t_l + \mu_h t_h - q\epsilon + \delta\mu_l(t_l - \epsilon) - \mu_h\kappa + (\chi_l^p + \delta)\mu_l\gamma_l + \mu_h\gamma_h\right) \\ & = (1 - \chi_l^e \mu_l + \delta\mu_l)[\chi_l^p \mu_l (w_l - t_l) + \mu_h(w_h - t_h) + q\epsilon + \delta\mu_l(w_l - t_l + \epsilon) + \mu_h\kappa \\ & \quad - (\chi_l^p + \delta)\mu_l\gamma_l - \mu_h\gamma_h], \end{aligned}$$

(The tax receipts have not changed.) or, more compactly,

$$\begin{aligned} & w_l - (t_l - \epsilon + \gamma_l) \\ & = \left((\chi_l^e - \delta)\mu_l\right)^{-1} \left((\chi_l^p + \delta)\mu_l(t_l - \epsilon + \gamma_l) + \mu_h(t_h - \epsilon - \kappa + \gamma_h)\right) \\ & = \left(1 - (\chi_l^e - \delta)\mu_l\right) \left[(\chi_l^p + \delta)\mu_l(w_l - (t_l - \epsilon + \gamma_l)) + \mu_h(w_h - (t_h - \epsilon - \kappa + \gamma_h))\right], \end{aligned}$$

or, even more compactly,

$$w_l - \hat{t}_l = \left(\hat{\chi}_l^e \mu_l\right)^{-1} \left(\hat{\chi}_l^p \mu_l \hat{t}_l + \mu_h \hat{t}_h\right) = \left(1 - \hat{\chi}_l^e \mu_l\right) \left[\hat{\chi}_l^p \mu_l (w_l - \hat{t}_l) + \mu_h(w_h - \hat{t}_h)\right],$$

where $\hat{\chi}_l^p = \chi_l^p + \delta$, $\hat{\chi}_l^e = \chi_l^e - \delta$, $\hat{\chi}_l^a = \chi_l^a = 0$, $\hat{t}_l = t_l - \epsilon + \gamma_l$, $\hat{t}_h = t_h - \epsilon - \kappa + \gamma_h$. Letting $\hat{w}_e = \left(\hat{\chi}_l^e \mu_l\right)^{-1} \left(\hat{\chi}_l^p \mu_l \hat{t}_l + \mu_h \hat{t}_h\right)$ and $\hat{\sigma} = (\hat{\chi}_l^p, \hat{\chi}_l^e, \hat{\chi}_l^a, 1, 0, 0, \hat{t}_l, \hat{t}_h, \hat{w}_e)$ gives $\varphi_l(\hat{\sigma}) = \hat{w}_e = \nu(\hat{\sigma})$. As $w_h - t_h \geq w_h - t_l - \phi$ by inequality (16), $w_h - \hat{t}_h \geq w_h - \hat{t}_l - \phi > w_l - \hat{t}_l$, is satisfied and the tax receipts are strictly positive. Therefore, $\hat{\sigma}$ satisfies all constraints of Problem (PP) but yields a higher objective function value than σ, because $\hat{\chi}_l^p = \chi_l^p + \delta > \chi_l^p$. ■

Lemma 4

Proof. Consider any solution σ to Problem (PP′). From constraint (20) it has to hold that $t_l + \phi \geq t_h$. Suppose for a contradiction that $t_l + \phi > t_h$, implying that $(1 - \theta p)(w_h - t_h) > (1 - \theta p)(w_h - t_l - \phi)$. Then, there exist an $\epsilon_h > 0$ and (as $\chi_l^p > 0$) an $\epsilon_l = -\frac{\mu_h}{\chi_l^p \mu_l}\epsilon_h < 0$ such that $t_l + \phi + \epsilon_l = t_h + \epsilon_h$, or

$$t_l + \phi = t_h + \left(1 + \frac{\mu_h}{\chi_l^p \mu_l}\right) \epsilon_h.$$

Let $\tilde{t}_i = t_i + \epsilon_i$. As $\tilde{t}_l + \phi = t_l + \phi + \epsilon_l = t_h + \epsilon_h = \tilde{t}_h$, as $\phi < (w_h - w_l)$,

$$(1 - \theta p)(w_h - \tilde{t}_h) = (1 - \theta p)(w_h - \tilde{t}_l - \phi) > (1 - \theta p)(w_l - \tilde{t}_l) \tag{29}$$

and, starting from $\varphi_l(\sigma) = w_e = \nu(\sigma)$, as agents of type l pay a lower tax, using $\epsilon_l = -\frac{\mu_h}{\chi_l^p \mu_l}\epsilon_h$,

$$\begin{aligned}
(1 - \theta p)(w_l - \tilde{t}_l) > w_e &= (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l \tilde{t}_l + \mu_h \tilde{t}_h) \\
&= (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l t_l + \mu_h t_h + \chi_l^p \mu_l \epsilon_l + \mu_h \epsilon_h) \\
&= (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l t_l + \mu_h t_h) \\
&= \nu(\sigma) \\
&= (1 - \chi_l^e \mu_l) \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_h))\right] \\
&= (1 - \chi_l^e \mu_l) \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_h) - \chi_l^p \mu_l \epsilon_l - \mu_h \epsilon_h\right] \\
&= (1 - \chi_l^e \mu_l) \left[\chi_l^p \mu_l (w_l - \tilde{t}_l) + \mu_h (w_h - \tilde{t}_h)\right].
\end{aligned}$$

Notice that the tax receipts implied by σ are bounded away from zero by $\nu(\sigma)$ as, by inequality (18), $w_h - t_h > w_l - t_l \geq 0$. In fact, due to $\varphi_l(\sigma) = w_e = \nu(\sigma) > 0$, $w_l - t_l > 0$. The last set of inequalities and equalities can be summarized as

$$(1 - \theta p)(w_l - \tilde{t}_l) > (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l \tilde{t}_l + \mu_h \tilde{t}_h) = (1 - \chi_l^e \mu_l) \left[\chi_l^p \mu_l (w_l - \tilde{t}_l) + \mu_h (w_h - \tilde{t}_h)\right] > 0.$$

Then, there exists a $\gamma > 0$, $\gamma < w_l - \tilde{t}_l$ ($< w_h - \tilde{t}_h$ by (29)), such that, letting $\bar{t}_i = \tilde{t}_i + \gamma$,

$$(1 - \theta p)(w_l - \bar{t}_l) > (\chi_l^e \mu_l)^{-1} (\chi_l^p \mu_l \bar{t}_l + \mu_h \bar{t}_h) > (1 - \chi_l^e \mu_l) \left[\chi_l^p \mu_l (w_l - \bar{t}_l) + \mu_h (w_h - \bar{t}_h)\right] > 0.$$

Then, there exists a $\delta_1 > 0$, $\delta_1 < \chi_l^e$, such that, using $\theta = 1 - \chi_l^e \mu_l$,

$$(1 - (1 - (\chi_l^e - \delta_1)\mu_l)p)(w_l - \bar{t}_l) > ((\chi_l^e - \delta_1)\mu_l)^{-1} ((\chi_l^p + \delta_1)\mu_l \bar{t}_l + \mu_h \bar{t}_h)$$

and a $\delta_2 > 0$, $\delta_2 < \chi_l^e$, such that

$$\begin{aligned}
&((\chi_l^e - \delta_2)\mu_l)^{-1} ((\chi_l^p + \delta_2)\mu_l \bar{t}_l + \mu_h \bar{t}_h) \\
&> (1 - (\chi_l^e - \delta_2)\mu_l) \left[(\chi_l^p + \delta_2)\mu_l (w_l - \bar{t}_l) + \mu_h (w_h - \bar{t}_h)\right] > 0.
\end{aligned}$$

Let $\delta = \min\{\delta_1, \delta_2\}$. Then,

$$
\begin{aligned}
(1-(1-(\chi_l^e-\delta)\mu_l)p)(w_l-\bar{t}_l) &> ((\chi_l^e-\delta)\mu_l)^{-1}\left((\chi_l^p+\delta)\mu_l\bar{t}_l+\mu_h\bar{t}_h\right) \\
&> (1-(\chi_l^e-\delta)\mu_l)\left[(\chi_l^p+\delta)\mu_l(w_l-\bar{t}_l)+\mu_h(w_h-\bar{t}_h)\right] > 0.
\end{aligned}
$$

Let $\hat{\chi}_l^p = \chi_l^p + \delta$, $\hat{\chi}_l^e = \chi_l^e - \delta$, $\hat{\chi}_l^a = \chi_l^a$, $\hat{p} = \hat{\chi}_l^a \mu_l = \chi_l^a \mu_l = p$, and $\hat{\theta} = (1 - \hat{\chi}_l^e \mu_l)$. Then, there exists an $\eta > 0$ such that, letting $\check{t}_i = \bar{t}_i - \eta$,

$$
(1-\hat{\theta}\hat{p})(w_l-\check{t}_l) > (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\check{t}_l+\mu_h\check{t}_h\right) = \hat{\theta}\left[\hat{\chi}_l^p\mu_l(w_l-\check{t}_l)+\mu_h(w_h-\check{t}_h)\right] > 0.
$$

Then, there are a $\kappa_l > 0$ and a $\kappa_h = -\frac{\hat{\chi}_l^p \mu_l}{\mu_h}\kappa_l < 0$ such that, letting $\hat{t}_i = \check{t}_i + \kappa_i$,

$$
\begin{aligned}
(1-\hat{\theta}\hat{p})(w_l-\hat{t}_l) &= (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\hat{t}_l+\mu_h\hat{t}_h\right) \\
&= (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\check{t}_l+\mu_h\check{t}_h+\hat{\chi}_l^p\mu_l\kappa_l+\mu_h\kappa_h\right) \\
&= (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\check{t}_l+\mu_h\check{t}_h\right) \\
&= \hat{\theta}\left[\hat{\chi}_l^p\mu_l(w_l-\check{t}_l)+\mu_h(w_h-\check{t}_h)\right] \\
&= \hat{\theta}\left[\hat{\chi}_l^p\mu_l(w_l-\check{t}_l)+\mu_h(w_h-\check{t}_h)-\hat{\chi}_l^p\mu_l\kappa_l-\mu_h\kappa_h\right] \\
&= \hat{\theta}\left[\hat{\chi}_l^p\mu_l(w_l-\hat{t}_l)+\mu_h(w_h-\hat{t}_h)\right] > 0,
\end{aligned}
$$

or, summarizing these equations,

$$
(1-\hat{\theta}\hat{p})(w_l-\hat{t}_l) = (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\hat{t}_l+\mu_h\hat{t}_h\right) = \hat{\theta}\left[\hat{\chi}_l^p\mu_l(w_l-\hat{t}_l)+\mu_h(w_h-\hat{t}_h)\right] > 0.
$$

Letting $\hat{w}_e = (\hat{\chi}_l^e\mu_l)^{-1}\left(\hat{\chi}_l^p\mu_l\hat{t}_l+\mu_h\hat{t}_h\right)$ and $\hat{\sigma} = (\hat{\chi}_l^p, \hat{\chi}_l^e, \hat{\chi}_l^a, 1, 0, 0, \hat{t}_l, \hat{t}_h, \hat{w}_e)$, the last equation implies $\varphi_l(\hat{\sigma}) = \hat{w}_e = \nu(\hat{\sigma})$. The tax receipts are still bounded away from zero. Since $t_l + \epsilon_l + \phi = t_h + \epsilon_h$, as established in the very beginning, and $\kappa_l > 0$ while $\kappa_h < 0$, it holds that $t_l + \epsilon_l + \gamma - \eta + \kappa_l + \phi > t_h + \epsilon_h + \gamma - \eta + \kappa_h$ so that $\hat{t}_l + \phi > \hat{t}_h$. Hence, constraint (20) is satisfied and $w_h - \hat{t}_h > w_l - \hat{t}_l$. Therefore, $\hat{\sigma}$ satisfies all constraints of Problem (PP′) but yields a higher objective function value than σ, because $\hat{\chi}_l^p = \chi_l^p + \delta > \chi_l^p$, a contradiction. ■

Proposition 5

Proof. Consider Problem (PP′). Following Lemmas 3 and 4, using (24) to replace p and χ_l^e, (23) to replace χ_l^a, and noting that $\chi_l^p, \chi_l^e > 0$, the solution to Problem (PP′) has to satisfy

$$t_h = t_l + \phi, \tag{30}$$

$$(1-\theta((1-\chi_l^p)\mu_l-(1-\theta)))(w_l-t_l) = w_e, \tag{31}$$

$$(1-\theta)w_e = \left(\chi_l^p\mu_l t_l+\mu_h t_h\right), \tag{32}$$

$$w_e = \theta \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_h)\right], \tag{33}$$

$$(1 - \chi_l^p)\mu_l - (1 - \theta) \geq 0. \tag{34}$$

Inequality (34) ensures that $\chi_l^a \mu_l \geq 0$. The objective function depends only on the choice of χ_l^p. I first show that a choice of χ_l^p implies a unique σ via equations (30)–(33). Given a choice of $\chi_l^p > 0$, equations (30)–(33) are four equations in four unknowns t_l, t_h, θ, and w_e that have to hold. Then, θ implies χ_l^e, which together with χ_l^p implies χ_l^a, completing σ. Equation (34), with θ being a function of χ_l^p, is then the only constraint to maximizing the objective via maximizing χ_l^p. Combining equations (31) and (32) each with equation (33) replacing t_h from equation (30) gives, since $\theta < 1$,

$$(1 - \theta((1 - \chi_l^p)\mu_l - (1 - \theta)))(w_l - t_l) = \theta \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)\right], \tag{35}$$

$$(1 - \theta)^{-1} \left(\chi_l^p \mu_l t_l + \mu_h t_l + \mu_h \phi\right) = \theta \left[\chi_l^p \mu_l (w_l - t_l) + \mu_h (w_h - t_l - \phi)\right]. \tag{36}$$

For later reference, from (36), with $t_h = t_l + \phi$ in mind, the total tax receipts collected are

$$\left(\chi_l^p \mu_l t_l + \mu_h t_h\right) = \frac{(1 - \theta)\theta}{1 + (1 - \theta)\theta} \left(\chi_l^p \mu_l w_l + \mu_h w_h\right), \tag{37}$$

so that, via (32), the wage in enforcement, w_e, can be expressed as

$$w_e = (1 - \theta)^{-1} \left(\chi_l^p \mu_l t_l + \mu_h t_h\right) = \frac{\theta}{1 + (1 - \theta)\theta} \left(\chi_l^p \mu_l w_l + \mu_h w_h\right). \tag{38}$$

Using $\mu_h(w_h - t_l - \phi) = \mu_h(w_h - w_l + w_l - t_l - \phi) = \mu_h(w_h - w_l - \phi) + \mu_h(w_l - t_l)$ in (35):

$$\begin{aligned}
&(1 - \theta((1 - \chi_l^p)\mu_l - (1 - \theta)))(w_l - t_l) = \theta(\chi_l^p \mu_l + \mu_h)(w_l - t_l) + \theta \mu_h (w_h - w_l - \phi) \\
\Leftrightarrow &(1 - \theta(\mu_l - \chi_l^p \mu_l - (1 - \theta) + \chi_l^p \mu_l + \mu_h))(w_l - t_l) = \theta \mu_h (w_h - w_l - \phi) \\
\Leftrightarrow &(1 - \theta^2)(w_l - t_l) = \theta \mu_h (w_h - w_l - \phi). \qquad (39)
\end{aligned}$$

Using the same expression in (36) gives

$$\begin{aligned}
&(\chi_l^p \mu_l t_l + \mu_h t_l + \mu_h \phi) = (1 - \theta)\theta(\chi_l^p \mu_l + \mu_h)(w_l - t_l) + (1 - \theta)\theta \mu_h (w_h - w_l - \phi) \\
\Leftrightarrow &(\chi_l^p \mu_l + \mu_h) t_l + \mu_h \phi = (1 - \theta)\theta(\chi_l^p \mu_l + \mu_h)(w_l - t_l) + (1 - \theta)\theta \mu_h (w_h - w_l - \phi) \\
&\qquad + (\chi_l^p \mu_l + \mu_h) w_l - (\chi_l^p \mu_l + \mu_h) w_l
\end{aligned}$$

$$\Leftrightarrow [1 + (1 - \theta)\theta](\chi_l^p \mu_l + \mu_h)(w_l - t_l) = (\chi_l^p \mu_l + \mu_h) w_l + \mu_h \phi - (1 - \theta)\theta \mu_h (w_h - w_l - \phi). \tag{40}$$

Given χ_l^p, equations (39) and (40) are two equations in the two unknowns t_l and θ. Rewriting both for an expression for $w_l - t_l$ and equalizing them gives

$$\frac{(\chi_l^p \mu_l + \mu_h) w_l + \mu_h \phi - (1-\theta)\theta\mu_h (w_h - w_l - \phi)}{[1 + (1-\theta)\theta](\chi_l^p \mu_l + \mu_h)} = \frac{\theta \mu_h (w_h - w_l - \phi)}{(1-\theta^2)}.$$

Rewriting gives

$$\theta\mu_h (w_h - w_l - \phi)\left[\frac{1+(1-\theta)\theta}{1-\theta^2}(\chi_l^p \mu_l + \mu_h) + (1-\theta)\right] - (\chi_l^p \mu_l + \mu_h) w_l - \mu_h \phi = 0. \tag{41}$$

Suppressing the arguments representing parameters, let $h : (0,1) \times [0,1] \to \mathbb{R}$ be given by

$$h(x, \chi_l^p) = x\mu_h (w_h - w_l - \phi)\left[\frac{1+(1-x)x}{1-x^2}(\chi_l^p \mu_l + \mu_h) + (1-x)\right] - (\chi_l^p \mu_l + \mu_h) w_l - \mu_h \phi.$$

Fixing $\chi_l^p \in [0,1]$, it follows that

$$\lim_{x \to 0} h(x, \chi_l^p) = -(\chi_l^p \mu_l + \mu_h) w_l - \mu_h \phi < 0, \quad \lim_{x \to 1} h(x, \chi_l^p) = +\infty > 0,$$

and

$$h_x(x, \chi_l^p) = \mu_h (w_h - w_l - \phi)\left((\chi_l^p \mu_l + \mu_h)\frac{1 + 2x - 2x^2 + x^4}{(1-x^2)^2} + 1 - 2x\right) > 0 \tag{42}$$

because

$$\mu_h > -\frac{1 - 2x - 2x^2 + 4x^3 + x^4 - 2x^5}{1 + 2x - 2x^2 + x^4}. \tag{43}$$

which in turn holds because the right hand side of (43) can be shown to be strictly less than 0.1 for all $x \in [0,1]$, while $\mu_h \geq 0.15$ by Assumption 1. Thus, for all $\chi_l^p \in [0,1]$, there exists a unique $\theta(\chi_l^p) \in (0,1)$ such that $h(\theta(\chi_l^p), \chi_l^p) = 0$. Still suppressing the dependence on parameters, the function $\theta : [0,1] \to (0,1)$ has a continuous derivative $\theta'(\chi_l^p) < \mu_l$ on the interior of its domain. Suppose for a contradiction that $\theta'(\chi_l^p) \geq \mu_l$. Fixing x at $\theta = \theta(\chi_l^p) \in (0,1)$, using the implicit function theorem, and rewriting $\theta'(\chi_l^p) = -\frac{h_{\chi_l^p}(x, \chi_l^p)}{h_x(x, \chi_l^p)}\Big|_{x=\theta} \geq \mu_l$ gives

$$\begin{aligned}
0 &\geq \mu_h (w_h - w_l - \phi)\left((\chi_l^p \mu_l + \mu_h)\frac{1 + 2\theta - 2\theta^2 + \theta^4}{(1-\theta^2)^2} + 1 - 2\theta\right) \\
&\quad + \left(\theta\mu_h (w_h - w_l - \phi)\frac{1 + (1-\theta)\theta}{1-\theta^2} - w_l\right) \\
&> \left(\theta\mu_h (w_h - w_l - \phi)\frac{1 + (1-\theta)\theta}{1-\theta^2} - w_l\right) + (1-\theta)\theta\mu_h (w_h - w_l - \phi) - \mu_h \phi \qquad (44)
\end{aligned}$$

$$\geq (\chi_l^p \mu_l + \mu_h)\left(\theta\mu_h(w_h - w_l - \phi)\frac{1+(1-\theta)\theta}{1-\theta^2} - w_l\right) + (1-\theta)\theta\mu_h(w_h - w_l - \phi) - \mu_h\phi$$
$$= h(\theta, \chi_l^p),$$

where the strict inequality holds because,

$$\begin{aligned}
&\mu_h(w_h - w_l - \phi)\left((\chi_l^p\mu_l + \mu_h)\frac{1+2\theta-2\theta^2+\theta^4}{(1-\theta^2)^2} + 1 - 2\theta\right) \\
&\geq \mu_h(w_h - w_l - \phi)\left(\mu_h\frac{1+2\theta-2\theta^2+\theta^4}{(1-\theta^2)^2} + 1 - 2\theta\right) \\
&> (1-\theta)\theta\mu_h(w_h - w_l - \phi) - \mu_h\phi
\end{aligned}$$

or

$$\mu_h(w_h - w_l - \phi)\left(\mu_h\frac{1+2\theta-2\theta^2+\theta^4}{(1-\theta^2)^2} + 1 - 2\theta - (1-\theta)\theta\right) > -\mu_h\phi$$

which holds since the right-hand side is negative and the left-hand side is nonnegative as

$$\mu_h \geq -\frac{1-3\theta-\theta^2+6\theta^3-\theta^4-3\theta^5+\theta^6}{1+2\theta-2\theta^2+\theta^4} \tag{45}$$

which in turn holds because the right hand side of (45) can be shown to be strictly less than 0.15 for all $\theta \in [0,1]$, while $\mu_h \geq 0.15$ by Assumption 1. Thus, inequality (44) contradicts $h(\theta, \chi_l^p) = 0$ as $\theta = \theta(\chi_l^p)$, establishing that $\theta'(\chi_l^p) < \mu_l$. Given $\theta = \theta(\chi_l^p) \in (0,1)$, it follows that $\chi_l^e = \mu_l^{-1}(1-\theta)$ and thus $\chi_l^a = 1 - \chi_l^p - \chi_l^e$; equations (39), (30), and (38) give t_l, t_h, and w_e, respectively. That is, a choice of χ_l^p implies a regime σ. The solution to problem (PP′) is σ^* implied by the maximal choice χ_l^{p*} that satisfies inequality (34). Still suppressing the dependence on the parameters, let $g : [0,1] \to \mathbb{R}$ be the rewritten inequality (34) at $\theta = \theta(\chi_l^p)$, so that, as $\mu_l + \mu_h = 1$,

$$g(\chi_l^p) = \theta(\chi_l^p) - (\chi_l^p\mu_l + \mu_h).$$

Then, χ_l^{p*} solves

$$\max_{\chi_l^p \in [0,1]} \chi_l^p \quad s.t. \quad g(\chi_l^p) \geq 0, \tag{46}$$

As $\theta'(\chi_l^p) < \mu_l$, it follows that $g'(\chi_l^p) = \theta'(\chi_l^p) - \mu_l < 0$. As $\theta(\chi_l^p) \in (0,1)$ for all $\chi_l^p \in [0,1]$, $g(1) = \theta(1) - 1 < 0$ and as $\mu_l w_l \geq \mu_h(w_h - w_l)$ by Assumption 1,

$$\begin{aligned} h(\mu_h, 0) &= \mu_h^2(w_h - w_l - \phi)\left[\frac{1+(1-\mu_h)\mu_h}{1-\mu_h^2}\mu_h + 1 - \mu_h\right] - \mu_h w_l - \mu_h\phi \\ &= \mu_h\left(\mu_h\frac{w_h - w_l - \phi}{1-\mu_h^2} - w_l - \phi\right) \\ &= \frac{\mu_h}{1-\mu_h^2}\Big(\mu_h(w_h - w_l - \phi) - (1-\mu_h^2)(w_l + \phi)\Big) \\ &< \frac{\mu_h}{1-\mu_h^2}\left(\mu_h(w_h - w_l) - \mu_l w_l\right) \leq 0 \end{aligned} \tag{47}$$

and $h_x(x, \chi_l^p) > 0$ for all $\chi_l^p \in [0,1]$, implying that $\theta(0) > \mu_h$, so that $g(0) = \theta(0) - \mu_h > 0$. It follows that χ_l^{p*} solves $g(\chi_l^p) = 0$ so that $\chi_l^{p*} > 0$ and $\theta^* = \theta(\chi_l^{p*})$ is given by

$$\theta^* = \theta(\chi_l^{p*}) = \chi_l^{p*}\mu_l + \mu_h. \tag{48}$$

Thus, $\chi_l^{e*} = 1 - \chi_l^{p*}$ and $\chi_l^{a*} = 0$, so that there are no appropriators and all agents that do not produce are employed in enforcement. Equation (48) and $h(\theta^*, \chi_l^{p*}) = h(\theta(\chi_l^{p*}), \chi_l^{p*}) = 0$ are two equations in the two unknowns θ^* and χ_l^{p*}. Using (48) in $h(\theta^*, \chi_l^{p*}) = 0$ and rewriting, θ^* solves $h^*(\theta) = 0$, where, suppressing parameters, $h^* : (0,1) \to \mathbb{R}$ is given by

$$h^*(\theta) = \frac{\theta}{1-\theta^2}\mu_h(w_h - w_l - \phi) - \theta w_l - \mu_h\phi, \tag{49}$$

which is a strictly convex function of θ with

$$\lim_{\theta\to 0} h^*(\theta) = -\mu_h\phi < 0, \quad \lim_{\theta\to 1} h^*(\theta) = \infty > 0.$$

Therefore, h^* has a unique root θ^*. As a higher ϕ decreases $h^*(\theta)$ for all θ, an increase in ϕ increases θ^*. Via (48), θ^* implies a unique χ_l^{p*} that is given by

$$\chi_l^{p*} = \mu_l^{-1}(\theta^* - \mu_h), \tag{50}$$

which implies a unique χ_l^{e*}, while $\chi_l^{a*} = 0$ as argued above, as well as a unique w_e^* given by

$$w_e^* = \frac{\theta^*}{1+(1-\theta^*)\theta^*}\left(\chi_l^{p*}\mu_l w_l + \mu_h w_h\right). \tag{51}$$

From (48) it further follows from (31) that $w_e^* = w_l - t_l^*$, implying a unique $t_l^* = w_l - w_e^*$ and thus $t_h^* = t_l^* + \phi$. Further, consider equation (51) and replace χ_l^{p*} using equation (50) so that

$$w_e^* = \frac{\theta^*}{1 + (1 - \theta^*)\theta^*} \left((\theta^* - \mu_h) w_l + \mu_h w_h\right), \tag{52}$$

which shows that w_e^* is strictly increasing in θ^*; the limits of w_e^* with θ^* approaching 0 and 1 are 0 and $\bar{w} > w_l$, respectively. Thus, there exists a $\hat{\theta}^* < 1$ such that $w_e^* = w_l$ at $\hat{\theta}^*$ and $w_e^* > w_l$ for all $\theta^* > \hat{\theta}^*$. As an increase in ϕ increases θ^*, for ϕ approaching $(w_h - w_l)$, from (49), θ^* has to approach 1 and thus increase beyond $\hat{\theta}^*$. It follows that for high enough ϕ, $t_l^* = w_l - w_e^* < 0$, which completes the proof. ■

Proposition 6

Proof. Consider equation (28). The left-hand side is a convex function, negative-valued for θ approaching zero and switching sign with θ approaching one, with a positive derivative at its unique root θ^*, and a higher ϕ decreases it for all θ. Therefore, θ^* increases with ϕ. Then, from equations (25), χ_l^{p*} increases, and χ_l^{e*} decreases. It follows from equations (27) that w_e^* increases, as it can be shown to increase in both θ^* and χ_l^{p*}, that t_l^* decreases because w_e^* increases, and that the difference $t_h^* - t_l^* = \phi$ increases with ϕ. ■

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