# What Would Legislators Do If They Could Act Unilaterally?*

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**Abstract**

I study an environment with two policy issues, two opportunistic parties, both ideological and reelection concerns, and four further assumptions: (1) The salient policy issue entails a conflict between higher- and lower-income citizens, while the other issue does not. (2) Voters vote based on expected policy alignment. (3) For higher-income citizens in office, group identity, or ideology, is sufficiently important relative to reelection concerns. (4) Parties select candidates from a set of higher- and lower-income citizens to maximize the probability of winning. Given these assumptions, the often-observed predominance of higher-income citizens in the national legislature implies that no legislator, not even lower-income citizens in office, would enact lower-income citizens' preferred policy if they could act unilaterally. This result is independent of policy outcomes and how they are determined. It contributes to the debate about the role of lower-income citizens' preferences in the policymaking process, suggesting it might be limited.

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# 1 Introduction

The question what role lower-income citizens' policy preferences play in the policymaking process in democracies is not settled.[1] Some argue that lower-income citizens' policy preferences are underrepresented (e.g., Gilens 2005, 2009; Carnes 2012; Gilens and Page 2014; Peters and Ensink 2015). Others disagree (e.g., Soroka and Wlezien 2008; Ura and Ellis 2008; Kelly and Enns 2010; Brunner et al. 2013; Branham et al. 2017). This paper highlights that under four assumptions, the often-observed predominance of higher-income citizens in the national legislature (e.g., Carnes 2012, 2018; Thompson et al. 2019; Gagliarducci et al. 2010; Dal Bó et al. 2017) has a stark implication. Lower-income citizens' policy preferences play a limited role in the policymaking process in the following sense. No legislator—not even lower-income citizens in office—would enact lower-income citizens' preferred redistribution policy if they could act unilaterally. This implication holds regardless of how policy outcomes are determined in the legislature, what policy outcomes arise, and what the legislature composition is beyond the predominance of higher-income citizens.

The four assumptions are as follows. (1) The salient policy issue entails a conflict between higher- and lower-income citizens, while most higher-income citizens agree with most lower-income citizens on another policy issue. This assumption is motivated by the debate mentioned above. If salient policy issues do not entail a conflict between higher- and lower-income citizens, then the role of lower-income citizens' policy preferences in the policymaking process should not be a concern. (2) Voters vote based on expected policy alignment in the sense that they vote for candidates whose expected choices of policies to enact if given an opportunity to act unilaterally they prefer. This assumption encapsulates the idea that voters care about their representatives' policy stances, and that their true policy stances of course can be thought of as the policies they would enact if they could unilaterally decide on policies. (3) For higher-income citizens in office, group identity, or ideology, is sufficiently important relative to reelection concerns. This assumption is quite weak in the sense that it only requires that, for higher-income citizens in office, ideology is not unimportant even if reelection concerns matter more. Given laws recently passed by the mostly higher-income-background members of the US Congress, for example, arguably, this assumption is not unreasonable.[2] (4) Regardless of the specific processes, barriers, and individual traits that determine the potential candidates parties consider suitable and select candidates from to maximize the probability of winning, both higher- and lower-income citizens are among them. This assumption amounts to it not being impossible altogether for candidates to be lower-income citizens because there are lower-income citizens that parties would be willing to select. One may interpret it as, e.g., some lower-income citizens possessing the charisma and political skills to win a party nomination.[3]

I embed these assumptions in an environment with two policy issues, opportunistic parties, ideological and reelection concerns facing legislators, and redistribution as the salient policy issue

[1] Throughout, higher and lower income should be thought of as moderately—not extremely—high and low incomes.

[2] For example, the 115th Congress' Public Law 97—the December 2017 tax reform—was expected to reduce redistribution. For details, see the law at https://www.congress.gov/bill/115th-congress/house-bill/1/text and the Congressional Budget Office's estimates of its distributional effects at https://www.cbo.gov/publication/53429.

[3] Carnes (2018) discusses possible reasons why it is not very common for US lower-income citizens to run for office.

(e.g., Meltzer and Richard 1981). The ideological and reelection concerns deliberately bias the environment against the main result. Allowing for two policy issues, one of which may or may not matter to citizens, ensures that the main result does not depend on whether or not there is a second policy dimension as such. I model multiple electoral districts and abstract from political processes in the legislature to focus on what policy legislators would enact if they could act unilaterally—which, arguably, is the policy they really support—rather than policy outcomes.

The environment respects several empirical observations. First, lower-income citizens—those with less-than-average income—constitute the majority in most electoral districts.[4] Second, lower-income citizens prefer more redistribution than higher-income citizens.[5] Third, voters do not expect lower-income candidates to be less effective in office than higher-income candidates (Carnes and Lupu 2016; Campbell and Cowley 2014).[6] Fourth, legislators' own policy preferences play an important role in determining their actions in office (Levitt 1996; Lee et al. 2004; Matsusaka 2026), and in the environment here, they determine what legislators would do if they could act unilaterally. Finally, holding office may pay an income premium. Such office-holding premia are well documented across many democratic societies (e.g., Gagliarducci et al. 2010; Eggers and Hainmueller 2009; Peichl et al. 2013; Kotakorpi et al. 2017; Berg 2020). These premia arise from relatively high legislator salaries, outside income while in office, and increased income potential in a post-legislature career.[7] With an outside-income interpretation, for example, the premium might be determined by restrictions on outside activity imposed by the political institutions. It might depend on explicit restrictions, implicit rules and custom, or the time commitment required of legislators. At least in the short run, these restrictions are exogenous from the point of view of individual legislators.

The main result provides a necessary condition for the predominance of higher-income citizens in the national legislature under the four stated assumptions. The analysis does not explain this predominance because it ignores equilibrium multiplicity, but since the main result is a necessary condition, equilibrium selection is not important for this paper. The necessary condition is that no legislator, not even lower-income citizens who hold office, would enact lower-income citizens' preferred redistribution policy if they could act unilaterally. This implication suggests that the role of lower-income citizens' redistribution preferences in the policymaking process might be limited.

While there is plenty of literature focusing on observed levels of redistribution, this paper has

[4]For example, in all congressional districts for the 115th United States Congress, median household income was less than mean household income (US Census Bureau, 2012–2016 ACS 5-year estimates, accessed on 4/20/2018).

[5]Studies of determinants of redistribution preferences—such as self-interest, efficiency concerns, risk aversion, inequality aversion, ideology, social identity, or the roles of effort and luck in determining incomes—consistently find that lower-income citizens prefer more redistribution than higher-income citizens (e.g., Corneo and Grüner 2002; Klor and Shayo 2010; Esarey et al. 2012; Durante et al. 2014; Lefgren et al. 2016; Gee et al. 2017; Tepe et al. 2021).

[6]While voters value experience in office (e.g, Lublin 1994; Squire 1995; Hobolt and Høyland 2011; Kendall et al. 2015), they do not seem to consider high income to indicate a candidate has the skills required to be an overall good representative. However, Mattozzi and Snowberg (2018) use convenience samples from Amazon's Mechanical Turk to argue that US voters might expect a link between private sector success and the ability to direct funds to the district.

[7]On high legislator salaries, see, e.g., Berg (2020). On outside income while in office, see, e.g., Gagliarducci et al. (2010); Eggers and Hainmueller (2009); Peichl et al. (2013); Geys and Mause (2013); Kotakorpi et al. (2017); Cirone et al. (2021); Weschle (2024); Dahlgaard et al. (2026). On post-legislature careers, see, e.g., Diermeier et al. (2005); Mattozzi and Merlo (2008); Eggers and Hainmueller (2009); Parker and Parker (2009); Palmer and Schneer (2016).

nothing to say about them because it focuses on an entirely different question.[8] It altogether abstracts from how policy outcomes are determined and what policy outcomes arise. The results do not speak to observed roll-call votes. These processes and observations do not matter for the paper's insight because the question it addresses is independent of them. For example, any policy outcome may arise if in some unmodeled process of legislative voting, legislators vote strategically, coalition building is required, or agenda-setting power matters. The main result holds regardless because none of these features of legislative voting affect what policy legislators would enact if they could act unilaterally—which again, arguably, is the policy they really support, no matter how they vote in the legislature—given the observed predominance of higher-income citizens among legislators. Of course, many legislators express support for redistributive policies in public statements or publicly recorded votes. However, public statements and votes might involve messaging to constituents if they do not affect the payoff-relevant policy outcome (Snyder and Ting 2003).

**Further related literature.** Providing a necessary condition for the predominance of higher-income citizens in national legislatures under a given set of assumptions, the paper concerns legislature composition and is related to the literature on politician selection and citizen candidates (Osborne and Slivinski 1996; Besley and Coate 1997). Focusing on quality, ability, or valence, see, e.g., Carrillo and Mariotti (2001), Galasso and Nannicini (2011), and Mattozzi and Merlo (2015) on the role of parties and Caselli and Morelli (2004), Messner and Polborn (2004), and Poutvaara and Takalo (2007) on the role of pay. These papers cannot speak to what legislators would do on redistribution if they could act unilaterally, given higher-income citizens' predominance.

I describe the environment in Section 2, analyze it in Section 3, discuss alternative channels, assumptions, and empirical tests in Section 4, and then conclude. All proofs are in Appendix A.

## 2 Model

There are an odd finite number $d > 1$ of equally-sized, pairwise disjoint electoral districts indexed by $j \in D = \{1, \dots, d\}$. Two parties $A$ and $B$ compete in $d$ two-candidate district elections to determine the $d$ legislators to represent the districts in a legislature. At the outset, parties $A$ and $B$ simultaneously select their candidate for each district election from the potential candidates. Then, citizens vote in their district's election to elect their representative, and the game ends.[9]

**Demographics.** There is a unit-measure continuum of risk-neutral citizens of four types based on their income and stance on regulation. Types are indexed by $(i, \chi) \in \{l, h\} \times \{0, 1\}$, where $i \in \{l, h\}$ indicates lower income, $w_l > 0$, or finite higher income, $w_h > w_l$, while $\chi \in \{0, 1\}$ indicates support for regulation, $\chi = 1$, or opposition to it, $\chi = 0$.[10] Each district $j$ has $\lambda^i_\chi \mu^j_i > 0$ citizens of each type

[8] On the roles of, e.g., issue bundling, rent seeking, targeted transfers, and campaign contributions in determining redistribution, see Roemer (1998), Rodríguez (2004), Fernández and Levy (2008), and Campante (2011), respectively.

[9] In particular, I abstract altogether from policy outcomes and a process for the legislature to arrive at them.

[10] Two income levels lend simplicity, but the analysis and results extend naturally to more than two income levels.

$(i, \chi)$ residing in it, where $\lambda_1^i > 0$ is the share of citizens with income $w_i$ who support regulation, $\lambda_0^i = 1 - \lambda_1^i > 0$ is the share of citizens with income $w_i$ who oppose regulation, $\mu_i^j > 0$ is the share of citizens with income $w_i$ in district $j$, and $\sum_{i,\chi} \lambda_\chi^i \mu_i^j = \sum_i \mu_i^j = 1/d$. Further, $\sum_j \mu_i^j = \mu_i$ for all $i \in \{l, h\}$, where $\mu_l + \mu_h = 1$. Median income is less than mean income $\bar{w} = \mu_l w_l + \mu_h w_h$: $\mu_l > 1/2 > \mu_h$. There are districts, collected in $D_l = \{j \in D : \mu_l^j > \mu_h^j\}$, in which the majority of citizens have lower income. There may be districts, collected in $D_h = \{j \in D : \mu_l^j < \mu_h^j\}$, in which the majority of citizens have higher income. In every district, either higher-income or lower-income citizens are a majority: $D = D_l \cup D_h$. However, in most districts, most citizens have lower income: $|D_l| > d/2 > |D_h|$. By Footnote 4, in the US, for example, $|D_l| = d > 0 = |D_h|$. Let $\hat{\imath}_j \in \{l, h\}$ and $-\hat{\imath}_j \in \{l, h\} \backslash \{\hat{\imath}_j\}$, respectively, indicate the majority and minority income groups in district $j$.

**Policies and mechanical voting.** The policy issues are redistribution $t \in \{0, 1\}$ and regulation $\delta \in \{0, 1\}$. First, every citizen, including legislators, has their income taxed proportionally at rate $t \in \{0, 1\}$ and receives a lump-sum transfer $\tau \geq 0$. The budget must balance, which with only $d$ legislators can be written as $\tau(\mu_l + \mu_h) = t(\mu_l w_l + \mu_h w_h)$, implying a transfer $\tau = t\bar{w}$. After-tax incomes equal before-tax incomes for $t = 0$ and the mean income for $t = 1$.[11] Second, citizens supporting regulation experience utility $\theta \geq 0$, while citizens opposing it experience disutility $-\theta$, if and only if regulation is enacted, $\delta = 1$. Allowing for regulation to either matter to citizens, $\theta > 0$, or not matter to citizens, $\theta = 0$, ensures that the main result does not depend on whether there is a second policy dimension as such. The payoffs of voters of type $(i, \chi)$ are

$$(1) \qquad \phi_{i,\chi}(t, \delta) = \begin{cases} \bar{w} + \delta\theta(-1)^{1-\chi} & \text{if } t = 1, \\ w_i + \delta\theta(-1)^{1-\chi} & \text{if } t = 0, \end{cases}$$

where $(-1)^{1-\chi}$ changes the sign of $\delta\theta$ for voters who oppose regulation. Redistribution is the salient policy issue, and most higher-income citizens agree with most lower-income citizens on regulation:

**Assumption 1.** *$\bar{w} - w_l > \theta$ and $\lambda_1^i > 1/2$ for all $i \in \{l, h\}$.*

The inequality $\bar{w} - w_l > \theta$ formalizes redistribution being salient in the sense that benefits related to regulation cannot fully compensate lower-income citizens for too little redistribution. This assumption also implies $w_h - \bar{w} > \theta$ so that regulation cannot compensate higher-income citizens for too much redistribution.[12] The inequalities $\lambda_1^h > 1/2$ and $\lambda_1^l > 1/2$ formalize the assumption that regulation does not entail a policy conflict between higher- and lower-income citizens because most higher-income citizens and most lower-income citizens support regulation.

In the two-candidate district election, voters vote based on policy alignment in the following

[11] The restriction to either full redistribution or no redistribution at all is without loss in the context of the analysis here. First, this stylized environment captures the observation that citizens with higher and lower incomes prefer different levels of redistribution. Second, the extent of redistribution the available policies enact is irrelevant for the analysis here as long as it differs. Third, allowing for a trade-off between the size of the pie and its distribution would not add anything material, but redistribution would be less than full and enacted with a tax rate less than one.

[12] As $\mu_l > \mu_h = 1 - \mu_l$: $w_h - \bar{w} = w_h - \mu_l w_l - \mu_h w_h = \mu_l(w_h - w_l) > \mu_h(w_h - w_l) = \mu_h w_h - (1 - \mu_l) w_l = \bar{w} - w_l$.

sense. Randomizing if indifferent, each voter mechanically votes for the candidate whose expected policy choices if they could act unilaterally offer the voter a higher payoff (1):

**Assumption 2.** *Type-$(i,\chi)$ voters maximize $\phi_{i,\chi}(t,\delta)$ given the $(t,\delta)$ candidates would enact if they could act unilaterally.*

Legislators give up their market income $w_i$ but, including the legislator salary, have income $v_i \geq w_i$ while in office, implying a nonnegative office-holding premium, as the empirical evidence suggests. Income in office from, e.g., outside activity may depend on skills and abilities associated with the legislator's income background. In addition, legislators from income group $i$ incur an ideological utility cost $\kappa_i \geq 0$ if and only if their original income group's preferred redistribution policy is not enacted. This cost represents a sense of income group identity.[13] It makes it more likely that, e.g., legislators from a lower-income background would enact redistribution if they could act unilaterally because it would be costly for them not to do so. Finally, legislators from income group $i$ incur a cost $\zeta_i \geq 0$ if and only if the preferred policy of the majority income group in the district they represent is not enacted. This cost represents reelection concerns, possibly due in part to the prospect of losing access to lucrative outside-income opportunities.[14] It makes it more likely that, e.g., legislators representing a district with a lower-income majority would enact redistribution if they could act unilaterally because it would be costly for them not to do so. Both ideological and reelection concerns deliberately bias the environment towards legislators enacting redistribution if they can act unilaterally and therefore against the main result. However, ideology is sufficiently important relative to reelection concerns for legislators from a higher-income background:

**Assumption 3.** *$\kappa_h \geq \zeta_h - \mu_l(w_h - w_l)$.*

The inequality $\kappa_h \geq \zeta_h - \mu_l(w_h - w_l)$ ensures that for higher-income citizens in office, the ideological cost of higher-income citizens' preferred redistribution policy not being enacted is at least as high as the cost of the preferred policy of the majority income group in the district they represent not being enacted minus some positive number related to income inequality.

To ensure that, given a policy and district, being selected by a party cannot make citizens worse off than not being selected, legislators from group $i$ experience office-holding utility benefits $\beta_i \geq \kappa_i + \zeta_i$ that derive from, e.g., ego rents from status or perks and cannot be taxed. These utility benefits do not affect legislators' payoff comparisons and thus the results—they purely serve internal consistency. The payoff of a legislator of type $(i,\chi)$ representing district $j$ and unilaterally enacting policies $t$ and $\delta$ is

$$\psi_{i,\chi}(t,\delta,j) = \begin{cases} \beta_i + \bar{w} + \delta\theta(-1)^{1-\chi} - \mathbb{1}_{i=h} \times \kappa_h & \text{if } t = 1, j \in D_l, \\ \beta_i + \bar{w} + \delta\theta(-1)^{1-\chi} - \mathbb{1}_{i=h} \times \kappa_h - \zeta_i & \text{if } t = 1, j \in D_h, \\ \beta_i + v_i + \delta\theta(-1)^{1-\chi} - \mathbb{1}_{i=l} \times \kappa_l - \zeta_i & \text{if } t = 0, j \in D_l, \\ \beta_i + v_i + \delta\theta(-1)^{1-\chi} - \mathbb{1}_{i=l} \times \kappa_l & \text{if } t = 0, j \in D_h, \end{cases} \tag{2}$$

[13] See, e.g., Lind (2007), Shayo (2009), and Klor and Shayo (2010) on group identity and redistribution preferences.
[14] For analyses of the role of reelection in its own right, see, e.g., Duggan (2000); Van Weelden (2013).

where $(-1)^{1-\chi}$ changes the sign of $\delta\theta$ for legislators who oppose regulation, while $\mathbb{1}_{i=h}$ and $\mathbb{1}_{i=l}$, respectively, equal one if $i = h$ and $i = l$ and zero otherwise to indicate when the ideological utility cost applies. Given $j \in D$ and $t \in \{0,1\}$, if $\theta > 0$, then any legislator, in line with their stance on regulation, would enact $\delta = \chi$ if they could act unilaterally because $\psi_{i,\chi}(t,\chi,j) > \psi_{i,\chi}(t,-\chi,j)$, where $-\chi \in \{0,1\} \setminus \{\chi\}$. If $\theta = 0$, then regulation does not matter to legislators or voters. Given $j \in D$ and $\delta \in \{0,1\}$, what legislators would do on redistribution if they could act unilaterally follows from comparing, respectively, the first and third entries and the second and fourth entries in (2). A legislator from income group $i$ who represents district $j$ and who can act unilaterally enacts redistribution policy $t = 1$ if and only if $\psi_{i,\chi}(1,\delta,j) > \psi_{i,\chi}(0,\delta,j)$.

**Strategic candidate selection.** From candidates' policy choices if they could act unilaterally and Assumption 2 follows voters' mechanical voting behavior. Given voters' mechanical voting, parties' candidate selections are the only strategic decisions. Parties simultaneously and independently across districts select candidates in each district from their potential candidates.[15] Let $\mathcal{S}_P$ denote the strategy set of party $P \in \{A,B\}$. A generic element $s_P = (s_{P,1},\ldots,s_{P,d}) \in \mathcal{S}_P$ is a collection of candidate selections for all districts, where $s_{P,j}$ indicates the citizen type of party $P$'s candidate in district $j$. Both parties' potential candidates in each district include citizens of all types:

**Assumption 4.** *$\mathcal{S}_P = \mathcal{S} \equiv (\{l,h\} \times \{0,1\})^d$ for all $P \in \{A,B\}$.*

That is, in all districts, both parties can select citizens from any income group and with any regulation preferences as their candidates. Letting $-P \in \{A,B\}\setminus\{P\}$, given a profile $(s_{P,j}, s_{-P,j})$ of district-$j$ candidate selections for both parties, $\pi_j(s_{P,j}, s_{-P,j})$ denotes the probability of party $P \in \{A,B\}$ winning the seat in district $j \in D$. Naturally, $\pi_j(s_{-P,j}, s_{P,j}) = 1 - \pi_j(s_{P,j}, s_{-P,j})$. These probabilities are discussed below. Party $P$'s objective is to maximize its expected number of seats in the legislature,[16]

$$V(s_P, s_{-P}) = \sum_{j \in D} \pi_j(s_{P,j}, s_{-P,j}).$$

**Definition 1.** *An equilibrium is a strategy profile $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that, for all $P \in \{A,B\}$,*

$$V(s_P^*, s_{-P}^*) \geq V(s_P, s_{-P}^*) \quad \forall s_P \in \mathcal{S}.$$

Finally, let $i_j^* \in \{l,h\}$ denote the income group of the legislator representing district $j \in D$. Let $\Delta_i = \{j \in D : i_j^* = i\}$ be the set of all districts represented by a legislator from income group $i \in \{l,h\}$.

**Definition 2.** *Income group $i \in \{l,h\}$ predominates in the legislature if and only if $|\Delta_i| > d/2$.*

[15] Parties can be thought of as bearing all campaign costs. For example, respectively, about 43%, 52%, and 69% of all candidates for the US House of Representatives in the 2016 election contributed or loaned \$0, no more than \$1,000, and no more than \$10,000 to their campaign (Federal Election Commission, accessed on 7/2/2019).

[16] I discuss alternative party objectives in Section 4.

# 3 Analysis

Given the district they represent, what legislators would do on redistribution if they could act unilaterally depends on how, respectively, the first and third entries and the second and fourth entries in (2) compare. Because ideology is sufficiently important relative to reelection concerns for legislators from a higher-income background, they would not enact more redistribution if they could act unilaterally regardless of the district they represent.

**Observation 1.** *For any* $\chi \in \{0, 1\}$, $\delta \in \{0, 1\}$, *and* $j \in D$, $\psi_{h,\chi}(0, \delta, j) \geq \psi_{h,\chi}(1, \delta, j)$.

When representing a district with a higher-income majority, legislators from a higher-income background who can act unilaterally can enact no redistribution without incurring ideological costs or being concerned about reelection. When representing a district with a lower-income majority, the ideological cost associated with enacting redistribution dominates the reelection concerns associated with not enacting redistribution.

For legislators from a lower-income background, what they would do on redistribution if they could act unilaterally depends on the district they represent and parameters.

**Observation 2.** *Fix any* $\chi \in \{0, 1\}$ *and* $\delta \in \{0, 1\}$. *If* $j \in D_l$, *then* $\psi_{l,\chi}(1, \delta, j) > \psi_{l,\chi}(0, \delta, j)$ *if and only if* $v_l < \bar{w} + \kappa_l + \zeta_l$. *If* $j \in D_h$, *then* $\psi_{l,\chi}(1, \delta, j) > \psi_{l,\chi}(0, \delta, j)$ *if and only if* $v_l < \bar{w} + \kappa_l - \zeta_l$.

Ideological concerns push lower-income citizens in office towards enacting redistribution if they can act unilaterally. Reelection concerns push them in the same direction if they represent a district with a lower-income majority and the opposite direction if they represent a district with a higher-income majority. Thus, in the former case, they would enact redistribution if they could act unilaterally unless their in-office income is high enough to not only induce them to individually prefer no redistribution but also overcome both their ideological and reelection concerns. In the latter case, for them to enact redistribution if they can act unilaterally, the in-office income together with their reelection concerns must outweigh both their income after redistribution and their ideological concerns.

The policies candidates are expected to enact if they could act unilaterally once in office determine voters' mechanical voting behavior, which in turn determines the probability of winning the election in district $j \in D$. For example, suppose $\theta > 0$ and consider a candidate of type $(h, 1)$. Since $\psi_{h,1}(t, 1, j) > \psi_{h,1}(t, 0, j)$ and, by Observation 1, $\psi_{h,1}(0, \delta, j) \geq \psi_{h,1}(1, \delta, j)$, if they could act unilaterally, they would choose to enact regulation, $\delta = 1$, and no redistribution, $t = 0$. By (1), a voter of type $(h, 0)$ associates payoff $\phi_{h,0}(0, 1) = w_h - \theta$ with voting for them, while a voter of type $(l, 1)$ associates payoff $\phi_{l,1}(0, 1) = w_l + \theta$ with voting for them. If both candidates in district $j \in D$ are of the same type, then all voters are indifferent among them and thus randomize. Each candidate is expected to receive the same share of votes, in which case a fair coin decides the election:

$$\pi_j(y, y) = 1/2 \quad \forall y \in \{l, h\} \times \{0, 1\}. \tag{3}$$

If the candidates are not of the same type, then there are three cases: $v_l < \bar{w} + \kappa_l - \zeta_l$; $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$; and $\bar{w} + \kappa_l + \zeta_l \leq v_l$. First, suppose $v_l < \bar{w} + \kappa_l - \zeta_l$. By Observations 1 and 2, legislators from the lower-income group would enact redistribution if they could act unilaterally, while legislators from the higher-income group would enact no redistribution if they could act unilaterally. Lower-income citizens' in-office income is too low to, together with their reelection concerns, outweigh both their income when redistribution is enacted and their ideological concerns when representing a district with a higher-income majority. It is therefore also too low to not only induce them to individually prefer no redistribution but also overcome both their ideological and reelection concerns when representing a district with a lower-income majority. Lemma 1 characterizes properties of the probability of winning the seat in district $j$.

**Lemma 1.** *Suppose $v_l < \bar{w} + \kappa_l - \zeta_l$. Fix any $j \in D$, $i' \in \{l, h\}$, and $\chi', \chi'' \in \{0, 1\}$.*

1. *If $\theta = 0$, then $\pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = 1$ and $\pi_j((i', \chi'), (i', \chi'')) = 1/2$.*

2. *If $\theta > 0$, then $\pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = 1$ and $\pi_j((i', 1), (i', 0)) = 1$.*

If regulation does not matter to citizens, $\theta = 0$, then voters always vote for a candidate from their income group, if there is one. Thus, a candidate from the district's majority income group wins with probability of at least one-half, while a candidate from the district's minority income group loses with probability of at least one-half. If regulation matters to citizens, $\theta > 0$, then the benefits related to it cannot fully compensate higher- and lower-income voters for too much or too little redistribution, respectively. If there are two candidates from the same income group but with different stances on regulation, then voters vote for the candidate who shares their preferences on it. Thus, a candidate from the district's majority income group who favors regulation, as do most voters in every district, wins the election with certainty unless their opponent is of the same type.

Second, suppose $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$. By Observation 1, legislators from the higher-income group would enact no redistribution if they could act unilaterally. However, by Observation 2, what legislators from the lower-income group would do if they could act unilaterally depends on the district they represent. On the one hand, representing a district with a lower-income majority, their in-office income is not high enough to induce them to individually prefer no redistribution and overcome both their ideological and reelection concerns. Thus, they would enact redistribution if they could act unilaterally. On the other hand, representing a district with a higher-income majority, together with their reelection concerns, their in-office income is high enough to outweigh both their income after redistribution and their ideological concerns. Thus, they would enact no redistribution if they could act unilaterally. Lemma 2 characterizes properties of the probability of winning the seat in district $j$.

**Lemma 2.** *Suppose $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$. Fix any $j \in D$, $i', i'' \in \{l, h\}$, and $\chi', \chi'' \in \{0, 1\}$.*

1. *If $\theta = 0$ and $j \in D_l$, then $\pi_j((l, \chi'), (h, \chi'')) = 1$ and $\pi_j((i', \chi'), (i', \chi'')) = 1/2$.*

2. *If $\theta = 0$ and $j \in D_h$, then $\pi_j((i', \chi'), (i'', \chi'')) = 1/2$.*

3. *If* $\theta > 0$ *and* $j \in D_l$, *then* $\pi_j((l, \chi'), (h, \chi'')) = 1$ *and* $\pi_j((i', 1), (i', 0)) = 1$.

4. *If* $\theta > 0$ *and* $j \in D_h$, *then* $\pi_j((i', 1), (i'', 0)) = 1$ *and* $\pi_j((i', \chi'), (i'', \chi')) = 1/2$.

Voters in a district with a higher-income majority are indifferent regarding the candidates' income backgrounds, but when regulation matters, $\theta > 0$, the majority prefers it be enacted. Thus, a candidate who favors regulation wins with certainty unless their opponent shares their stance. If regulation does not matter, $\theta = 0$, then voters in a district with a higher-income majority are always indifferent among candidates. In a district with a lower-income majority, that majority prefers a lower-income candidate, regardless of candidates' stances on regulation. Thus, any lower-income candidate wins against any higher-income opponent and ties with any lower-income opponent. If regulation matters, then between two candidates who only differ in their stances on regulation, the candidate who favors regulation, as do most voters in every district, wins with certainty.

Finally, suppose $\bar{w} + \kappa_l + \zeta_l \leq v_l$. Since $\bar{w} + \kappa_l - \zeta_l \leq \bar{w} + \kappa_l + \zeta_l \leq v_l$, by Observations 1 and 2, not only legislators with a higher-income background but also legislators with a lower-income background would enact no redistribution if they could act unilaterally regardless of the district they represent. Lower-income citizens' in-office income is high enough to not only induce them to individually prefer no redistribution but also overcome ideological and reelection concerns they might face. Therefore, voters evaluate candidates based on their stances on regulation alone. Lemma 3 characterizes the probability of winning the seat in district $j$.

**Lemma 3.** *Suppose* $\bar{w} + \kappa_l + \zeta_l \leq v_l$. *Fix any* $j \in D$, $i', i'' \in \{l, h\}$, *and* $\chi', \chi'' \in \{0, 1\}$.

1. *If* $\theta = 0$, *then* $\pi_j((i', \chi'), (i'', \chi'')) = 1/2$.

2. *If* $\theta > 0$, *then* $\pi_j((i', 1), (i'', 0)) = 1$ *and* $\pi_j((i', \chi'), (i'', \chi')) = 1/2$.

If regulation matters to citizens and exactly one candidate favors it, then that candidate wins the election with certainty, regardless of both candidates' income backgrounds. In all other cases, all voters in the district are indifferent among both candidates and thus randomize so that a fair coin flip is expected to decide the election.

**Proposition 1.** *An equilibrium exists. If* $|\Delta_h| > d/2$ *in equilibrium, then* $\psi_{i,\chi}(0, \delta, j) \geq \psi_{i,\chi}(1, \delta, j)$ *for all* $i \in \{l, h\}$ *and* $j \in D$.

This result states a necessary condition for higher-income citizens to predominate in the legislature. Not only legislators from a higher-income background but also legislators from a lower-income background must enact no redistribution if they could act unilaterally. The result follows from two insights. First, higher-income citizens do not predominate in the legislature in equilibrium if lower-income citizens who represent a district with a lower-income majority would still enact redistribution if they could act unilaterally. Their ideological concerns push lower-income citizens who hold office towards enacting more redistribution if they could act unilaterally regardless of the district they represent. Reelection concerns amplify this push if they represent a district with a lower-income majority but weaken it if they represent a district with a higher-income majority.

Lower-income citizens who hold office representing a district with a lower-income majority thus are most likely to still enact redistribution if they could act unilaterally. In those districts, as long as lower-income candidates would still enact redistribution if they could act unilaterally once in office, lower-income voters—the majority—vote for a lower-income candidate if there is one. Both parties therefore select lower-income citizens as their candidates in all districts with a lower-income majority, whose representatives thus are lower-income citizens. As most districts have a lower-income majority, most legislators are lower-income citizens.

Second, higher-income citizens predominate in the legislature in some equilibrium if lower-income citizens who hold office would enact no redistribution if they could act unilaterally regardless of the district they represent. In this case, voters are indifferent regarding candidates' income backgrounds. If regulation does not matter to citizens, then voters randomize their vote irrespective of what types the candidates in their district are. Every profile of candidate selections is an equilibrium. On the other hand, if regulation matters, then most voters in every district vote for a candidate who supports regulation irrespective of their income background, if there is one. Every profile of candidate selections from among the citizens who support regulation is an equilibrium. In either case, for example, a profile of candidate selections in which all candidates support regulation and both candidates in most districts are higher-income citizens is an equilibrium. In such an equilibrium, while many legislators may be lower-income citizens, most legislators are higher-income citizens.

Combining both insights, if higher-income citizens predominate in the legislature in equilibrium, then lower-income citizens who hold office must enact no redistribution if they could act unilaterally regardless of the district they represent. Lower-income citizens' in-office income must be high enough to not only induce them to individually prefer no redistribution but also overcome any ideological and reelection concerns they might have. Only in this case is there an equilibrium in which higher-income citizens predominate in the legislature. For this logic, it is irrelevant why such an equilibrium arises and not a different one. The relevant prediction is unambiguous: if higher-income citizens predominate in the legislature, then lower-income citizens in office, and there may be many, would enact no redistribution if they could act unilaterally, as would higher-income citizens. This insight applies regardless of how many legislators actually have a lower-income background. Even the extreme case of all legislators being higher-income citizens can arise only if lower-income citizens, once in office, would not enact more redistribution if they could act unilaterally.

Finally, any policy outcome can arise through any unmodeled process at the legislative stage without affecting the main result's insights because the question, analysis, and main result are entirely independent of what happens in the legislature.

## 4 Discussion

In this section, I discuss alternative channels, assumptions, and requirements for empirical tests.

**Alternative channels.** The result does not rest on the channel. An alternative channel involves social pressure as touched on by Krugman (2010). For example, candidates and office holders often interact with celebrities and wealthy donors. This experience may instill a sense of closeness with and social pressure by them in lower-income citizens, in which case, once in office, they would not enact more redistribution if they could act unilaterally. Without this sense of closeness, lower-income citizens would still enact more redistribution if they could act unilaterally and thus win most seats in the legislature. The predominance of higher-income citizens thus requires lower-income citizens to develop this sense of closeness and, again, not enact more redistribution if they could act unilaterally.

**Electoral advantages.** Voters might derive some additional benefit from their representative being a higher-income citizen due to, e.g., specific skills they bring. The results are unaffected as long as these additional benefits together with their preferred regulation policy do not fully compensate lower-income citizens for too little redistribution. In this case, if lower-income citizens in office would still enact more redistribution if they could act unilaterally, then they win most seats in the legislature. Higher-income citizens' predominance among legislators thus still requires lower-income citizens in office to not enact redistribution if they could act unilaterally. Only then do higher-income citizens have an electoral advantage, and they would win all seats.

**Campaign finance.** I implicitly assume that campaign costs are borne by the parties. If campaign costs had to be covered by candidates personally, then only wealthy or higher-income citizens could run for office. However, candidates rarely contribute substantially to their own campaigns. For example, respectively, about 43%, 52%, and 69% of all candidates for the US House of Representatives in the 2016 election contributed or loaned nothing at all, no more than $1,000, and no more than $10,000 to their campaign. Similarly, of all winners, respectively, about 83%, 86%, and 89% contributed or loaned nothing at all, no more than $1,000, and no more than $10,000 to their campaign. Of all nonincumbent winners, respectively, about 27%, 38%, and 45% contributed or loaned nothing at all, no more than $1,000, and no more than $10,000 to their campaign.[17] That is, high income or wealth is not necessary to win the office, let alone run for it. Specifically, candidates might raise money from interest groups that support candidates who are aligned with them on their issue.[18] Suppose running for office is prohibitively costly for individual citizens, but an interest group with abundant resources that opposes regulation is ready to finance the campaigns of all candidates who are aligned with them and would not enact regulation if they could act unilaterally. While citizens who support regulation cannot run for office, the main result is unaffected.

**Party objectives and discipline.** Two opportunistic parties selecting candidates ensures that an equilibrium exists without further restrictions on the income distribution.[19] The results are

[17] Data: Federal Election Commission, accessed on 7/2/2019.

[18] I focus on campaign finance (also see, e.g., Besley and Coate 2003, 2008). On lobbying, see, e.g., Besley and Coate (2001); Felli and Merlo (2006); Gehlbach et al. (2010).

[19] On the role of parties in their own right, see, e.g., Bernhardt et al. 2009; Galasso and Nannicini 2011.

unaffected if the two parties have opposing redistribution preferences and aim to select candidates who, once in office, are aligned with them and would enact the party's preferred redistribution policy if they could act unilaterally. Ignoring weakly dominated strategies, if lower-income citizens in office would still enact redistribution if they could act unilaterally, then the party favoring redistribution selects lower-income citizens in districts with a majority of lower-income citizens. Lower-income citizens thus win the seat in most districts, and higher-income citizens do not predominate in the legislature. If lower-income citizens in office would enact no redistribution if they could act unilaterally, then parties are indifferent regarding the income background of possible candidates, and higher-income citizens can predominate in the legislature in equilibrium. The predominance of higher-income citizens thus still requires lower-income citizens in office to not enact redistribution if they could act unilaterally. In addition, party discipline cannot be effective when legislator can act unilaterally. If the party favoring redistribution can compel lower-income citizens in office to enact it if they can act unilaterally, then higher-income citizens cannot predominate in the legislature.[20]

**Empirical tests.** While it is usually unobservable what a legislator would do if they could act unilaterally, the results remains testable. However, there are several requirements for an appropriate test. First, a legislator essentially can act unilaterally if their vote is pivotal in the legislature because, in this case, their vote decides the policy outcome. Therefore, for example, redistribution must be a salient policy issue, and the national legislature must decide about more redistribution by a margin of one vote. Also, the vote of some legislator from a lower-income background must count towards the majority to be pivotal. Depending on the precise setting, for example, a second chamber might need to reasonably be expected to pass the bill with certainty. Only then is a vote that is expected to decide whether the bill passes in the first chamber also expected to decide whether the bill passes overall. Yet, in such a scenario, one can assess what legislators from a lower-income background did in a situation in which they essentially acted unilaterally.

## 5 Conclusion

This paper highlights a possible implication of the often-observed predominance of higher-income citizens in the national legislature in democracies under four assumptions. These assumptions are that (1) redistribution is the salient policy issue, (2) voters vote for candidates based on expected policy alignment with them, (3) for higher-income citizens in office, ideology is sufficiently important relative to reelection concerns, and (4) parties select candidates from some set of higher- and lower-income citizens to maximize the probability of winning. Under these arguably reasonable assumptions, a necessary condition for higher-income citizens' predominance is that, regardless of income background, once in office, no citizen would enact lower-income citizens' preferred redistribution policy if they could act unilaterally. Lower-income citizens' redistribution preferences thus might play a limited role in the policymaking process. This implication holds regardless of policy

[20] Here, the implicit assumption is that party discipline cannot induce legislators from a higher-income background to enact redistribution if they can act unilaterally. Arguably, this assumption is not unreasonable.

outcomes, which are not studied or relevant for the insight, and legislature composition beyond higher-income citizens' predominance. Since the result is a necessary condition for higher-income citizens' predominance in equilibrium, a characterization of legislature composition that deals with the multiplicity of equilibria is not important for this paper's insights.

Future work could further explore the robustness of and alternative channels for the underlying logic and what assumptions might be violated empirically. For example, one might ask under what conditions this implication still arises with proportional representation or when the role of party politics or gatekeepers at the candidate selection stage is modeled more explicitly. One could similarly ask what role status and group identity derived from holding a prestigious office might play. Finally, one might ask how salient redistribution is as a policy issue in congressional elections.

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# A Appendix - Proofs

### Observation 1

*Proof.* If $j \in D_l$, then, since $\kappa_h \geq \zeta_h - \mu_l(w_h - w_l)$, $v_h \geq w_h$, $\mu_l + \mu_h = 1$, and $\bar{w} = \mu_l w_l + \mu_h w_h$, it follows that $\psi_{h,\chi}(0,\delta,j) = \beta_h + v_h + \delta\theta(-1)^{1-\chi} - \zeta_h \geq \beta_h + w_h + \delta\theta(-1)^{1-\chi} - \mu_l(w_h - w_l) - \kappa_h = \beta_h + \mu_l w_l + \mu_h w_h + \delta\theta(-1)^{1-\chi} - \kappa_h = \beta_h + \bar{w} + \delta\theta(-1)^{1-\chi} - \kappa_h = \psi_{h,\chi}(1,\delta,j)$.

If $j \in D_h$, then, since $v_h \geq w_h > \bar{w}$, $\kappa_h \geq 0$, and $\zeta_h \geq 0$, it follows that $\psi_{h,\chi}(0,\delta,j) = \beta_h + v_h + \delta\theta(-1)^{1-\chi} > \beta_h + \bar{w} + \delta\theta(-1)^{1-\chi} \geq \beta_h + \bar{w} + \delta\theta(-1)^{1-\chi} - \kappa_h - \zeta_h = \psi_{h,\chi}(1,\delta,j)$. ■

### Observation 2

*Proof.* Suppose $j \in D_l$. Then $\psi_{l,\chi}(1,\delta,j) > \psi_{l,\chi}(0,\delta,j) \iff \beta_l + \bar{w} + \delta\theta(-1)^{1-\chi} > \beta_l + v_l + \delta\theta(-1)^{1-\chi} - \kappa_l - \zeta_l \iff v_l < \bar{w} + \kappa_l + \zeta_l$. Suppose $j \in D_h$. Then $\psi_{l,\chi}(1,\delta,j) > \psi_{l,\chi}(0,\delta,j) \iff \beta_l + \bar{w} + \delta\theta(-1)^{1-\chi} - \zeta_l > \beta_l + v_l + \delta\theta(-1)^{1-\chi} - \kappa_l \iff v_l < \bar{w} + \kappa_l - \zeta_l$. ■

### Lemma 1

*Proof.* Suppose $v_l < \bar{w} + \kappa_l - \zeta_l$. By Observations 1 and 2, if they could act unilaterally, legislators of type $l$ would enact $t = 1$, while legislators of type $h$ would enact $t = 0$. Fix any $j \in D$, $i' \in \{l, h\}$, and $\chi', \chi'' \in \{0, 1\}$. Fix any $P \in \{A, B\}$.

1. Suppose $\theta = 0$. Suppose $i' = l$ and $s_{P,j} = (i', \chi') = (l, \chi')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1, 0) = \bar{w}$, while $s_{-P,j} = (i', \chi'') = (l, \chi'')$, which voters of type $(i, \chi)$ also associate with payoff $\phi_{i,\chi}(1, 0) = \bar{w}$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i', \chi'), (i', \chi'')) = \pi_j((l, \chi'), (l, \chi'')) = 1/2$. Similarly, suppose $i' = h$ and $s_{P,j} = (i', \chi') = (h, \chi')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0, 0) = w_i$, while $s_{-P,j} = (i', \chi'') = (h, \chi'')$, which voters of type $(i, \chi)$ also associate with payoff $\phi_{i,\chi}(0, 0) = w_i$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i', \chi'), (i', \chi'')) = \pi_j((h, \chi'), (h, \chi'')) = 1/2$. Therefore, $\pi_j((i', \chi'), (i', \chi'')) = 1/2$.

   Suppose $j \in D_l$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_l^j > \mu_h^j$ follows that $\mu_l^j > (1/d)/2$. Suppose $s_{P,j} = (\hat{i}_j, \chi') = (l, \chi')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1, 0) = \bar{w}$, while $s_{-P,j} = (-\hat{i}_j, \chi'') = (h, \chi'')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0, 0) = w_i$. Then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1, 0) = \bar{w} > w_l = \phi_{l,1}(0, 0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l, 0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1, 0) = \bar{w} > w_l = \phi_{l,0}(0, 0)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = \pi_j((l, \chi'), (h, \chi'')) = 1$. Similarly, suppose $j \in D_h$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_h^j > \mu_l^j$ follows that $\mu_h^j > (1/d)/2$. Suppose $s_{P,j} = (\hat{i}_j, \chi') = (h, \chi')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0, 0) = w_i$, while $s_{-P,j} = (-\hat{i}_j, \chi'') = (l, \chi'')$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1, 0) = \bar{w}$.

Then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{h,1}(0,0) = w_h > \bar{w} = \phi_{h,1}(1,0)$; similarly, $\lambda_0^h \mu_h^j$ voters of type $(h,0)$ vote for $s_{P,j}$ because $\phi_{h,0}(0,0) = w_h > \bar{w} = \phi_{h,0}(1,0)$. Since $\lambda_1^h \mu_h^j + \lambda_0^h \mu_h^j = \mu_h^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = \pi_j((h,\chi'), (l,\chi'')) = 1$. Therefore, $\pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = 1$.

2. Suppose $\theta > 0$. Suppose $i' = l$ and $s_{P,j} = (i',1) = (l,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,1)$, while $s_{-P,j} = (i',0) = (l,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$. Then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ and $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{i,1}(1,1) = \bar{w} + \theta > \bar{w} = \phi_{i,1}(1,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\}\mu_l^j + \min\{\lambda_1^l, \lambda_1^h\}\mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',1),(i',0)) = \pi_j((l,1),(l,0)) = 1$. Similarly, suppose $i' = h$ and $s_{P,j} = (i',1) = (h,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, while $s_{-P,j} = (i',0) = (h,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ and $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{i,1}(0,1) = w_i + \theta > w_i = \phi_{i,1}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\}\mu_l^j + \min\{\lambda_1^l, \lambda_1^h\}\mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',1),(i',0)) = \pi_j((h,1),(h,0)) = 1$. Therefore, $\pi_j((i',1),(i',0)) = 1$.

   Suppose $j \in D_l$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_l^j > \mu_h^j$ follows that $\mu_l^j > (1/d)/2$. Suppose $s_{P,j} = (\hat{i}_j, 1) = (l,1)$. Voters of type $(i,\chi)$ associate payoff $\phi_{i,\chi}(1,1)$ with $s_{P,j}$. If $s_{-P,j} = (-\hat{i}_j, 1) = (h,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,1) = \bar{w} + \theta > w_l + \theta = \phi_{l,1}(0,1)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l,0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,1) = \bar{w} - \theta > w_l - \theta = \phi_{l,0}(0,1)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, 1), (-\hat{i}_j, 1)) = \pi_j((l,1),(h,1)) = 1$. If $s_{-P,j} = (-\hat{i}_j, 0) = (h,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$, then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,1) = \bar{w} + \theta > w_l = \phi_{l,1}(0,0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l,0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,1) = \bar{w} - \theta > w_l = \phi_{l,0}(0,0)$ since $\bar{w} - w_l > \theta$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, 1), (-\hat{i}_j, 0)) = \pi_j((l,1),(h,0)) = 1$. Similarly, suppose $s_{P,j} = (\hat{i}_j, 0) = (l,0)$. Voters of type $(i,\chi)$ associate payoff $\phi_{i,\chi}(1,0) = \bar{w}$ with $s_{P,j}$. If $s_{-P,j} = (-\hat{i}_j, 1) = (h,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,0) = \bar{w} > w_l + \theta = \phi_{l,1}(0,1)$ since $\bar{w} - w_l > \theta$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l,0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,0) = \bar{w} > w_l - \theta = \phi_{l,0}(0,1)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, 0), (-\hat{i}_j, 1)) = \pi_j((l,0),(h,1)) = 1$. If $s_{-P,j} = (-\hat{i}_j, 0) = (h,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$, then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,0) = \bar{w} > w_l = \phi_{l,1}(0,0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l,0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,0) = \bar{w} > w_l = \phi_{l,0}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j, 0), (-\hat{i}_j, 0)) = \pi_j((l,0),(h,0)) = 1$. Therefore, $\pi_j((\hat{i}_j, \chi'), (-\hat{i}_j, \chi'')) = \pi_j((l,\chi'),(h,\chi'')) = 1$.

   Suppose $j \in D_h$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_h^j > \mu_l^j$ follows that $\mu_h^j > (1/d)/2$. Suppose $s_{P,j} = (\hat{i}_j, 1) = (h,1)$. Voters of type $(i,\chi)$ associate payoff $\phi_{i,\chi}(0,1)$ with $s_{P,j}$. If $s_{-P,j} = (-\hat{i}_j, 1) = (l,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,1)$, then $\lambda_1^h \mu_h^j$

voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{h,1}(0,1) = w_h + \theta > \bar{w} + \theta = \phi_{h,1}(1,1)$; similarly, $\lambda_0^h \mu_h^j$ voters of type $(h,0)$ vote for $s_{P,j}$ because $\phi_{h,0}(0,1) = w_h - \theta > \bar{w} - \theta = \phi_{h,0}(1,1)$. Since $\lambda_1^h \mu_h^j + \lambda_0^h \mu_h^j = \mu_h^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j,1),(-\hat{i}_j,1)) = \pi_j((h,1),(l,1)) = 1$. If $s_{-P,j} = (-\hat{i}_j,0) = (l,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$, then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{h,1}(0,1) = w_h + \theta > \bar{w} = \phi_{h,1}(1,0)$; similarly, $\lambda_0^h \mu_h^j$ voters of type $(h,0)$ vote for $s_{P,j}$ because $\phi_{h,0}(0,1) = w_h - \theta > \bar{w} = \phi_{h,0}(1,0)$ since $w_h - \bar{w} > \theta$. Since $\lambda_1^h \mu_h^j + \lambda_0^h \mu_h^j = \mu_h^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j,1),(-\hat{i}_j,0)) = \pi_j((h,1),(l,0)) = 1$. Similarly, suppose $s_{P,j} = (\hat{i}_j,0) = (h,0)$. Voters of type $(i,\chi)$ associate payoff $\phi_{i,\chi}(0,0) = w_i$ with $s_{P,j}$. If $s_{-P,j} = (-\hat{i}_j,1) = (l,1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,1)$, then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{h,1}(0,0) = w_h > \bar{w} + \theta = \phi_{h,1}(1,1)$ since $w_h - \bar{w} > \theta$; similarly, $\lambda_0^h \mu_h^j$ voters of type $(h,0)$ vote for $s_{P,j}$ because $\phi_{h,0}(0,0) = w_h > \bar{w} - \theta = \phi_{h,0}(1,1)$. Since $\lambda_1^h \mu_h^j + \lambda_0^h \mu_h^j = \mu_h^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j,0),(-\hat{i}_j,1)) = \pi_j((h,0),(l,1)) = 1$. If $s_{-P,j} = (-\hat{i}_j,0) = (l,0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$, then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ vote for $s_{P,j}$ because $\phi_{h,1}(0,0) = w_h > \bar{w} = \phi_{h,1}(1,0)$; similarly, $\lambda_0^h \mu_h^j$ voters of type $(h,0)$ vote for $s_{P,j}$ because $\phi_{h,0}(0,0) = w_h > \bar{w} = \phi_{h,0}(1,0)$. Since $\lambda_1^h \mu_h^j + \lambda_0^h \mu_h^j = \mu_h^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((\hat{i}_j,0),(-\hat{i}_j,0)) = \pi_j((h,0),(l,0)) = 1$. Therefore, $\pi_j((\hat{i}_j,\chi'),(-\hat{i}_j,\chi'')) = \pi_j((h,\chi'),(l,\chi'')) = 1$.

Combining the cases of $j \in D_l$ and $j \in D_h$, $\pi_j((\hat{i}_j,\chi'),(-\hat{i}_j,\chi'')) = 1$.

■

**Lemma 2**

*Proof.* Suppose $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$. By Observation 1, if they could act unilaterally, legislators of type $h$ would enact $t = 0$. By Observation 2, if they could act unilaterally, legislators of type $l$ representing district $j \in D$ would enact $t = 1$ if $j \in D_l$ and $t = 0$ if $j \in D_h$. Fix any $j \in D$, $i', i'' \in \{l,h\}$, and $\chi', \chi'' \in \{0,1\}$. Fix any $P \in \{A,B\}$.

1. Suppose $\theta = 0$ and $j \in D_l$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_l^j > \mu_h^j$ follows that $\mu_l^j > (1/d)/2$. Suppose $s_{P,j} = (l,\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$, while $s_{-P,j} = (h,\chi'')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,0) = \bar{w} > w_l = \phi_{l,1}(0,0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l,0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,0) = \bar{w} > w_l = \phi_{l,0}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((l,\chi'),(h,\chi'')) = 1$.

   Suppose $i' = l$ and $s_{P,j} = (i',\chi') = (l,\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$, while $s_{-P,j} = (i',\chi'') = (l,\chi'')$, which voters of type $(i,\chi)$ also associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',\chi'),(i',\chi'')) = \pi_j((l,\chi'),(l,\chi'')) = 1/2$. Similarly, suppose $i' = h$ and $s_{P,j} = (i',\chi') = (h,\chi')$, which voters of type $(i,\chi)$ associate with payoff

$\phi_{i,\chi}(0,0) = w_i$, while $s_{-P,j} = (i', \chi'') = (h, \chi'')$, which voters of type $(i, \chi)$ also associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i', \chi'), (i', \chi'')) = \pi_j((h, \chi'), (h, \chi'')) = 1/2$. Therefore, $\pi_j((i', \chi'), (i', \chi'')) = 1/2$.

2. Suppose $\theta = 0$ and $j \in D_h$. Given the profile $(s_{P,j}, s_{-P,j}) = ((i', \chi'), (i'', \chi''))$, voters of all types $(i, \chi)$ associate payoff $\phi_{i,\chi}(0,0) = w_i$ with both candidates and thus are indifferent. All voters randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i', \chi'), (i'', \chi'')) = 1/2$.

3. Suppose $\theta > 0$ and $j \in D_l$. From $\mu_l^j + \mu_h^j = 1/d$ and $\mu_l^j > \mu_h^j$ follows that $\mu_l^j > (1/d)/2$. Suppose $\chi' = 1$ and $s_{P,j} = (l, \chi') = (l, 1)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1,1)$. If $\chi'' = 1$ and $s_{-P,j} = (h, \chi'') = (h, 1)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,1) = \bar{w} + \theta > w_l + \theta = \phi_{l,1}(0,1)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l, 0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,1) = \bar{w} - \theta > w_l - \theta = \phi_{l,0}(0,1)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((l, \chi'), (h, \chi'')) = \pi_j((l, 1), (h, 1)) = 1$. If $\chi'' = 0$ and $s_{-P,j} = (h, \chi'') = (h, 0)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$, then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,1) = \bar{w} + \theta > w_l = \phi_{l,1}(0,0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l, 0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,1) = \bar{w} - \theta > w_l = \phi_{l,0}(0,0)$ since $\bar{w} - w_l > \theta$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((l, \chi'), (h, \chi'')) = \pi_j((l, 1), (h, 0)) = 1$. Similarly, suppose $\chi' = 0$ and $s_{P,j} = (l, \chi') = (l, 0)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$. If $\chi'' = 1$ and $s_{-P,j} = (h, \chi'') = (h, 1)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,0) = \bar{w} > w_l + \theta = \phi_{l,1}(0,1)$ since $\bar{w} - w_l > \theta$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l, 0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,0) = \bar{w} > w_l - \theta = \phi_{l,0}(0,1)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((l, \chi'), (h, \chi'')) = \pi_j((l, 0), (h, 1)) = 1$. If $\chi'' = 0$ and $s_{-P,j} = (h, \chi'') = (h, 0)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$, then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ vote for $s_{P,j}$ because $\phi_{l,1}(1,0) = \bar{w} > w_l = \phi_{l,1}(0,0)$; similarly, $\lambda_0^l \mu_l^j$ voters of type $(l, 0)$ vote for $s_{P,j}$ because $\phi_{l,0}(1,0) = \bar{w} > w_l = \phi_{l,0}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_0^l \mu_l^j = \mu_l^j > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((l, \chi'), (h, \chi'')) = \pi_j((l, 0), (h, 0)) = 1$. Therefore, $\pi_j((l, \chi'), (h, \chi'')) = 1$.

   Suppose $i' = l$ and $s_{P,j} = (i', 1) = (l, 1)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1,1)$, while $s_{-P,j} = (i', 0) = (l, 0)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(1,0) = \bar{w}$. Then $\lambda_1^l \mu_l^j$ voters of type $(l, 1)$ and $\lambda_1^h \mu_h^j$ voters of type $(h, 1)$ vote for $s_{P,j}$ because $\phi_{i,1}(1,1) = \bar{w} + \theta > \bar{w} = \phi_{i,1}(1,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\} \mu_l^j + \min\{\lambda_1^l, \lambda_1^h\} \mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i', 1), (i', 0)) = \pi_j((l, 1), (l, 0)) = 1$. Similarly, suppose $i' = h$ and $s_{P,j} = (i', 1) = (h, 1)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, while $s_{-P,j} = (i', 0) = (h, 0)$, which voters of type $(i, \chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ and $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{i,1}(0,1) = w_i + \theta > w_i = \phi_{i,1}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\}\mu_l^j + \min\{\lambda_1^l, \lambda_1^h\}\mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',1),(i',0)) = \pi_j((h,1),(h,0)) = 1$. Therefore, $\pi_j((i',1),(i',0)) = 1$.

4. Suppose $\theta > 0$ and $j \in D_h$. Suppose $s_{P,j} = (i',1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, while $s_{-P,j} = (i'',0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ and $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{i,1}(0,1) = w_i + \theta > w_i = \phi_{i,1}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\}\mu_l^j + \min\{\lambda_1^l, \lambda_1^h\}\mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',1),(i'',0)) = 1$.

   Suppose $s_{P,j} = (i',\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,\chi')$, while $s_{-P,j} = (i'',\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,\chi')$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',\chi'),(i'',\chi')) = 1/2$.

■

**Lemma 3**

*Proof.* Suppose $\bar{w} + \kappa_l + \zeta_l \leq v_l$. By Observations 1 and 2, if they could act unilaterally, both legislators of type $l$ and type $h$ would enact $t = 0$. Fix any $j \in D$, $i', i'' \in \{l,h\}$, and $\chi', \chi'' \in \{0,1\}$. Fix any $P \in \{A,B\}$.

1. Suppose $\theta = 0$. Given the profile $(s_{P,j}, s_{-P,j}) = ((i',\chi'),(i'',\chi''))$, voters of all types $(i,\chi)$ associate payoff $\phi_{i,\chi}(0,0) = w_i$ with both candidates and thus are indifferent. All voters randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',\chi'),(i'',\chi'')) = 1/2$.

2. Suppose $\theta > 0$. Suppose $s_{P,j} = (i',1)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,1)$, while $s_{-P,j} = (i'',0)$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,0) = w_i$. Then $\lambda_1^h \mu_h^j$ voters of type $(h,1)$ and $\lambda_1^l \mu_l^j$ voters of type $(l,1)$ vote for $s_{P,j}$ because $\phi_{i,1}(0,1) = w_i + \theta > w_i = \phi_{i,1}(0,0)$. Since $\lambda_1^l \mu_l^j + \lambda_1^h \mu_h^j \geq \min\{\lambda_1^l, \lambda_1^h\}\mu_l^j + \min\{\lambda_1^l, \lambda_1^h\}\mu_h^j = \min\{\lambda_1^l, \lambda_1^h\}(\mu_l^j + \mu_h^j) = \min\{\lambda_1^l, \lambda_1^h\}(1/d) > (1/d)/2$, $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',1),(i'',0)) = 1$.

   Suppose $s_{P,j} = (i',\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,\chi')$, while $s_{-P,j} = (i'',\chi')$, which voters of type $(i,\chi)$ associate with payoff $\phi_{i,\chi}(0,\chi')$. Then all voters are indifferent and randomize, each candidate is expected to receive the same share of votes, in which case a fair coin decides the election, and $\pi_j(s_{P,j}, s_{-P,j}) = \pi_j((i',\chi'),(i'',\chi')) = 1/2$.

■

**Proposition 1**

*Proof.* To establish the result, I first show that:

1. If $v_l < \bar{w} + \kappa_l - \zeta_l$, then, for all $\theta \geq 0$, an equilibrium exists, and in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) \in (\{\hat{i}_j\} \times \{0,1\})^2$ and $i_j^* = \hat{i}_j$ for all $j \in D$ and $|\Delta_h| < d/2$.
2. If $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$, then, for all $\theta \geq 0$, an equilibrium exists, and in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) \in (\{l\} \times \{0,1\})^2$ and $i_j^* = l$ for all $j \in D_l$ and $|\Delta_h| < d/2$.
3. If $\bar{w} + \kappa_l + \zeta_l \leq v_l$, then, for all $\theta \geq 0$, every $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that for all $j \in D$, $(s_{A,j}^*, s_{B,j}^*) = ((i_{A,j}, 1), (i_{B,j}, 1))$, $(i_{A,j}, i_{B,j}) \in \{l, h\}^2$, is an equilibrium. In some equilibrium, $|\Delta_h| > d/2$.

Consider each case in turn.

1. Suppose $v_l < \bar{w} + \kappa_l - \zeta_l$. There are two cases: $\theta = 0$ and $\theta > 0$.

   Suppose $\theta = 0$. Consider any $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that $(s_{A,j}^*, s_{B,j}^*) \in (\{\hat{i}_j\} \times \{0,1\})^2$ for all $j \in D$. By Lemma 1.1, $\pi_j(s_{A,j}^*, s_{B,j}^*) = 1/2$ and $\pi_j(s_{B,j}^*, s_{A,j}^*) = 1 - \pi_j(s_{A,j}^*, s_{B,j}^*) = 1/2$ for all $j \in D$. Party $P$'s expected payoff is

   $$V(s_P^*, s_{-P}^*) = \sum_{j \in D} \pi_j(s_{P,j}^*, s_{-P,j}^*) = \frac{1}{2}d.$$

   Consider any $P \in \{A, B\}$ and any $s_P' \in \mathcal{S}$, $s_P' \neq s_P^*$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s_{P,k}' \neq s_{P,k}^*$. Moreover, for all $k \in D'$, while $s_{-P,k}^* \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$, either $s_{P,k}' \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$ or $s_{P,k}' \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$. By Lemma 1.1, $\pi_k(s_{P,k}', s_{-P,k}^*) = 1/2$ in the former case and $\pi_k(s_{P,k}', s_{-P,k}^*) = 1 - \pi_k(s_{-P,k}^*, s_{P,k}') = 1 - 1 = 0$ in the latter case. Thus, $\pi_k(s_{P,k}', s_{-P,k}^*) \leq 1/2$ for all $k \in D'$, while $\pi_j(s_{P,j}', s_{-P,j}^*) = \pi_j(s_{P,j}^*, s_{-P,j}^*) = 1/2$ for all $j \in D \setminus D'$. Thus, letting $d' = |D'| > 0$, party $P$'s expected payoff is

   $$\begin{aligned} V(s_P', s_{-P}^*) = \sum_{j \in D} \pi_j(s_{P,j}', s_{-P,j}^*) &= \sum_{j \in D \setminus D'} \pi_j(s_{P,j}', s_{-P,j}^*) + \sum_{k \in D'} \pi_k(s_{P,k}', s_{-P,k}^*) \\ &= \frac{1}{2}(d - d') + \sum_{k \in D'} \pi_k(s_{P,k}', s_{-P,k}^*) \\ &\leq \frac{1}{2}(d - d') + \frac{1}{2}d' \\ &= V(s_P^*, s_{-P}^*). \end{aligned}$$

   Thus, $(s_A^*, s_B^*)$ is an equilibrium.

   Consider any $(s_A, s_B) \in \mathcal{S}^2$ such that $(s_{A,k}, s_{B,k}) \notin (\{\hat{i}_k\} \times \{0,1\})^2$ for some $k \in D$. There are two cases: either *(a)* for some $P \in \{A, B\}$, $s_{P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$ and $s_{-P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$; or *(b)* for some $P \in \{A, B\}$, $s_{P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$, while $s_{-P,k} \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$. Consider each case in turn.

*Case (a).* If $s_{P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$ and $s_{-P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1/2$ by Lemma 1.1 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \frac{1}{2} + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider any $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$. As $s_{-P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$, $\pi_k(s'_{P,k}, s_{-P,k}) = 1$ by Lemma 1.1 and

$$\begin{aligned} V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) &= 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) \\ &> V(s_P, s_{-P}). \end{aligned}$$

Thus, $(s_A, s_B)$ is not an equilibrium.

*Case (b).* If $s_{P,k} \in \{(-\hat{i}_k, 1), (-\hat{i}_k, 0)\}$ and $s_{-P,k} \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1 - \pi_k(s_{-P,k}, s_{P,k}) = 1 - 1 = 0$ by Lemma 1.1 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider any $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$. As $s_{-P,k} \in \{(\hat{i}_k, 1), (\hat{i}_k, 0)\}$, $\pi_k(s'_{P,k}, s_{-P,k}) = 1/2$ by Lemma 1.1 and

$$\begin{aligned} V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) &= \frac{1}{2} + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) \\ &> V(s_P, s_{-P}). \end{aligned}$$

Thus, $(s_A, s_B)$ is not an equilibrium.

Therefore, in every equilibrium $(s^*_A, s^*_B)$, $(s^*_{A,j}, s^*_{B,j}) \in (\{\hat{i}_j\} \times \{0, 1\})^2$ for all $j \in D$.

Suppose $\theta > 0$. Consider $(s^*_A, s^*_B) \in \mathcal{S}^2$ such that $(s^*_{A,j}, s^*_{B,j}) = ((\hat{i}_j, 1), (\hat{i}_j, 1))$ for all $j \in D$. By (3), $\pi_j(s^*_{P,j}, s^*_{-P,j}) = \pi_j((\hat{i}_j, 1), (\hat{i}_j, 1)) = 1/2$ for all $P \in \{A, B\}$ and $j \in D$. Party $P$'s expected payoff is

$$V(s^*_P, s^*_{-P}) = \sum_{j \in D} \pi_j(s^*_{P,j}, s^*_{-P,j}) = \frac{1}{2} d.$$

Consider any $P \in \{A, B\}$ and any $s'_P \in \mathcal{S}$, $s'_P \neq s^*_P$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s'_{P,k} \neq s^*_{P,k}$. Moreover, for all $k \in D'$, $s'_{P,k} \neq (\hat{i}_k, 1)$, while $s^*_{-P,k} = (\hat{i}_k, 1)$. By Lemma 1.2, $\pi_k(s'_{P,k}, s^*_{-P,k}) = 1 - \pi_k(s^*_{-P,k}, s'_{P,k}) = 1 - \pi_k((\hat{i}_k, 1), s'_{P,k}) = 1 - 1 = 0$ for all $k \in D'$, while $\pi_j(s'_{P,j}, s^*_{-P,j}) = \pi_j(s^*_{P,j}, s^*_{-P,j}) = 1/2$ for all $j \in D \setminus D'$. Thus, letting $d' = |D'| > 0$, party $P$'s expected payoff is

$$V(s'_P, s^*_{-P}) = \sum_{j \in D} \pi_j(s'_{P,j}, s^*_{-P,j}) = \frac{1}{2}(d - d') < \frac{1}{2} d = V(s^*_P, s^*_{-P}).$$

Thus, $(s_A^*, s_B^*)$ is an equilibrium.

Consider any $(s_A, s_B) \in \mathcal{S}^2$, $(s_A, s_B) \neq (s_A^*, s_B^*)$. That is, for some $k \in D$, $(s_{A,k}, s_{B,k}) \neq ((\hat{i}_k, 1), (\hat{i}_k, 1))$. There are two cases: either *(a)* for some $P \in \{A, B\}$, $s_{P,k} \neq (\hat{i}_k, 1)$, $s_{-P,k} \neq (\hat{i}_k, 1)$, and $\pi_k(s_{P,k}, s_{-P,k}) < 1$; or *(b)* for some $P \in \{A, B\}$, $s_{P,k} \neq (\hat{i}_k, 1)$, while $s_{-P,k} = (\hat{i}_k, 1)$. Consider each case in turn.

*Case (a).* If $s_{P,k} \neq (\hat{i}_k, 1)$, $s_{-P,k} \neq (\hat{i}_k, 1)$, and $\pi_k(s_{P,k}, s_{-P,k}) < 1$ for some $P \in \{A, B\}$, then

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) < 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} = (\hat{i}_k, 1)$. As $s_{-P,k} \neq (\hat{i}_k, 1)$, $\pi_k(s'_{P,k}, s_{-P,k}) = \pi_k((\hat{i}_k, 1), s_{-P,k}) = 1$ by Lemma 1.2 so that

$$V(s'_P, s_{-P}) = 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) > V(s_P, s_{-P}).$$

Thus, $(s_A, s_B)$ is not an equilibrium.

*Case (b).* If $s_{P,k} \neq (\hat{i}_k, 1)$ and $s_{-P,k} = (\hat{i}_k, 1)$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1 - \pi_k(s_{-P,k}, s_{P,k}) = 1 - \pi_k((\hat{i}_k, 1), s_{P,k}) = 1 - 1 = 0$ by Lemma 1.2 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} = (\hat{i}_k, 1)$. As $s_{-P,k} = (\hat{i}_k, 1)$, $\pi_k(s'_{P,k}, s_{-P,k}) = \pi_k((\hat{i}_k, 1), (\hat{i}_k, 1)) = 1/2$ by (3) so that

$$V(s'_P, s_{-P}) = \frac{1}{2} + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) > \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = V(s_P, s_{-P}).$$

Thus, $(s_A, s_B)$ is not an equilibrium.

Therefore, $(s_A^*, s_B^*)$ such that $(s_{A,j}^*, s_{B,j}^*) = ((\hat{i}_j, 1), (\hat{i}_j, 1))$ for all $j \in D$ is the unique equilibrium.

It follows that for all $\theta \geq 0$, an equilibrium exists, and in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) \in (\{\hat{i}_j\} \times \{0, 1\})^2$ and $i_j^* = \hat{i}_j$ for all $j \in D$, implying $\Delta_i = D_i$ for all $i \in \{l, h\}$ and thus $|\Delta_l| = |D_l| > d/2$ and $|\Delta_h| = |D_h| < d/2$.

2. Suppose $\bar{w} + \kappa_l - \zeta_l \leq v_l < \bar{w} + \kappa_l + \zeta_l$. There are two cases: $\theta = 0$ and $\theta > 0$.

   Suppose $\theta = 0$. Consider any $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that $(s_{A,j}^*, s_{B,j}^*) \in (\{\hat{i}_j\} \times \{0, 1\})^2$ for all $j \in D$. By Lemma 2.1, $\pi_j(s_{P,j}^*, s_{-P,j}^*) = 1/2$ for all $j \in D_l$. By Lemma 2.2, $\pi_j(s_{P,j}^*, s_{-P,j}^*) = 1/2$ for all $j \in D_h$. Party $P$'s expected payoff is

$$V(s_P^*, s_{-P}^*) = \sum_{j \in D} \pi_j(s_{P,j}^*, s_{-P,j}^*) = \frac{1}{2} d.$$

Consider any $P \in \{A, B\}$ and any $s'_P \in \mathcal{S}$, $s'_P \neq s^*_P$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s'_{P,k} \neq s^*_{P,k}$. Moreover, for all $k \in D'$, while $s^*_{-P,k} \in \{(\hat{\imath}_k, 1), (\hat{\imath}_k, 0)\}$, either $s'_{P,k} \in \{(\hat{\imath}_k, 1), (\hat{\imath}_k, 0)\}$ or $s'_{P,k} \in \{(-\hat{\imath}_k, 1), (-\hat{\imath}_k, 0)\}$. Thus, if $k \in D_l$, then by Lemma 2.1, $\pi_k(s'_{P,k}, s^*_{-P,k}) = 1/2$ in the former case and $\pi_k(s'_{P,k}, s^*_{-P,k}) = 1 - \pi_k(s^*_{-P,k}, s'_{P,k}) = 1 - 1 = 0$ in the latter case; if $k \in D_h$, then by Lemma 2.2, $\pi_k(s'_{P,k}, s^*_{-P,k}) = 1/2$. That is, $\pi_k(s'_{P,k}, s^*_{-P,k}) \leq 1/2 = \pi_k(s^*_{P,k}, s^*_{-P,k})$ for all $k \in D'$, while $\pi_j(s'_{P,j}, s^*_{-P,j}) = \pi_j(s^*_{P,j}, s^*_{-P,j})$ for all $j \in D \setminus D'$. Therefore, party $P$'s expected payoff is

$$V(s'_P, s^*_{-P}) = \sum_{j \in D} \pi_j(s'_{P,j}, s^*_{-P,j}) \leq \sum_{j \in D} \pi_j(s^*_{P,j}, s^*_{-P,j}) = V(s^*_P, s^*_{-P}).$$

Thus, $(s^*_A, s^*_B)$ is an equilibrium.

Consider any $(s_A, s_B) \in \mathcal{S}^2$ such that $(s_{A,k}, s_{B,k}) \notin (\{l\} \times \{0, 1\})^2$ for some $k \in D_l$. There are two cases: either *(a)* for some $P \in \{A, B\}$, $s_{P,k} \in \{(h, 1), (h, 0)\}$ and $s_{-P,k} \in \{(h, 1), (h, 0)\}$; or *(b)* for some $P \in \{A, B\}$, $s_{P,k} \in \{(h, 1), (h, 0)\}$, while $s_{-P,k} \in \{(l, 1), (l, 0)\}$. Consider each case in turn.

*Case (a).* If $s_{P,k} \in \{(h, 1), (h, 0)\}$ and $s_{-P,k} \in \{(h, 1), (h, 0)\}$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1/2$ by Lemma 2.1 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \frac{1}{2} + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider any $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} \in \{(l, 1), (l, 0)\}$. As $s_{-P,k} \in \{(h, 1), (h, 0)\}$, $\pi_k(s'_{P,k}, s_{-P,k}) = 1$ by Lemma 2.1 and

$$\begin{aligned} V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) &= 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) \\ &> V(s_P, s_{-P}). \end{aligned}$$

Thus, $(s_A, s_B)$ is not an equilibrium.

*Case (b).* If $s_{P,k} \in \{(h, 1), (h, 0)\}$ and $s_{-P,k} \in \{(l, 1), (l, 0)\}$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1 - \pi_k(s_{-P,k}, s_{P,k}) = 1 - 1 = 0$ by Lemma 2.1 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider any $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} \in \{(l, 1), (l, 0)\}$. As $s_{-P,k} \in \{(l, 1), (l, 0)\}$, $\pi_k(s'_{P,k}, s_{-P,k}) = 1/2$ by Lemma 2.1 and

$$\begin{aligned} V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) &= \frac{1}{2} + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) \\ &> V(s_P, s_{-P}). \end{aligned}$$

Thus, $(s_A, s_B)$ is not an equilibrium.

Therefore, in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) \in (\{l\} \times \{0,1\})^2$ for all $j \in D_l$.

Suppose $\theta > 0$. Consider $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that $(s_{A,j}^*, s_{B,j}^*) = ((\hat{i}_j, 1), (\hat{i}_j, 1))$ for all $j \in D$. By (3), $\pi_j(s_{P,j}^*, s_{-P,j}^*) = 1/2$ for all $j \in D$. Party $P$ has expected payoff

$$V(s_P^*, s_{-P}^*) = \sum_{j \in D} \pi_j(s_{P,j}^*, s_{-P,j}^*) = \frac{1}{2}d.$$

Consider any $P \in \{A, B\}$ and any $s'_P \in \mathcal{S}$, $s'_P \neq s_P^*$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s'_{P,k} \neq s_{P,k}^*$. Moreover, for all $k \in D'$, $s'_{P,k} \neq (\hat{i}_k, 1)$, while $s_{-P,k}^* = (\hat{i}_k, 1)$. Thus, if $k \in D_l$, then $\pi_k(s'_{P,k}, s_{-P,k}^*) = 1 - \pi_k(s_{-P,k}^*, s'_{P,k}) = 1 - 1 = 0$ by Lemma 2.3; if $k \in D_h$, then $\pi_k(s'_{P,k}, s_{-P,k}^*) = 1 - \pi_k(s_{-P,k}^*, s'_{P,k}) \leq 1 - 1/2 = 1/2$ by Lemma 2.4. That is, $\pi_k(s'_{P,k}, s_{-P,k}^*) \leq 1/2 = \pi_k(s_{P,k}^*, s_{-P,k}^*)$ for all $k \in D'$, while $\pi_j(s'_{P,j}, s_{-P,j}^*) = \pi_j(s_{P,j}^*, s_{-P,j}^*)$ for all $j \in D \setminus D'$. Therefore, party $P$'s expected payoff is

$$V(s'_P, s_{-P}^*) = \sum_{j \in D} \pi_j(s'_{P,j}, s_{-P,j}^*) \leq \sum_{j \in D} \pi_j(s_{P,j}^*, s_{-P,j}^*) = V(s_P^*, s_{-P}^*).$$

Thus, $(s_A^*, s_B^*)$ is an equilibrium.

Consider any $(s_A, s_B) \in \mathcal{S}^2$ such that $(s_{A,k}, s_{B,k}) \neq ((l,1),(l,1))$ for some $k \in D_l$. There are two cases: either *(a)* for some $P \in \{A, B\}$, $s_{P,k} \neq (l,1)$, $s_{-P,k} \neq (l,1)$, and $\pi_k(s_{P,k}, s_{-P,k}) < 1$; or *(b)* for some $P \in \{A, B\}$, $s_{P,k} \neq (l,1)$, while $s_{-P,k} = (l,1)$. Consider each case in turn.

*Case (a).* If $s_{P,k} \neq (l,1)$, $s_{-P,k} \neq (l,1)$, and $\pi_k(s_{P,k}, s_{-P,k}) < 1$ for some $P \in \{A, B\}$, then

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) < 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} = (l,1)$. As $s_{-P,k} \neq (l,1)$, $\pi_k(s'_{P,k}, s_{-P,k}) = 1$ by Lemma 2.3 and

$$\begin{aligned} V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) &= 1 + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) \\ &> V(s_P, s_{-P}). \end{aligned}$$

Thus, $(s_A, s_B)$ is not an equilibrium.

*Case (b).* If $s_{P,k} \neq (l,1)$ and $s_{-P,k} = (l,1)$ for some $P \in \{A, B\}$, then $\pi_k(s_{P,k}, s_{-P,k}) = 1 - \pi_k(s_{-P,k}, s_{P,k}) = 1 - 1 = 0$ by Lemma 2.3 and

$$V(s_P, s_{-P}) = \pi_k(s_{P,k}, s_{-P,k}) + \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}) = \sum_{j \in D, j \neq k} \pi_j(s_{P,j}, s_{-P,j}).$$

Consider $s'_P \in \mathcal{S}$ such that $s'_{P,j} = s_{P,j}$ for all $j \in D, j \neq k$ and $s'_{P,k} = (l,1)$. As $s_{-P,k} = (l,1)$,

$\pi_k(s'_{P,k}, s_{-P,k}) = \pi_k((l,1),(l,1)) = 1/2$ by (3) and

$$V(s'_P, s_{-P}) = \pi_k(s'_{P,k}, s_{-P,k}) + \sum_{j\in D, j\neq k} \pi_j(s_{P,j}, s_{-P,j}) = \frac{1}{2} + \sum_{j\in D, j\neq k} \pi_j(s_{P,j}, s_{-P,j})$$
$$> V(s_P, s_{-P}).$$

Thus, $(s_A, s_B)$ is not an equilibrium.

Therefore, in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) = ((l,1),(l,1))$ for all $j \in D_l$.

It follows that for all $\theta \geq 0$, an equilibrium exists, and in every equilibrium $(s_A^*, s_B^*)$, $(s_{A,j}^*, s_{B,j}^*) \in (\{l\} \times \{0,1\})^2$ and $i_j^* = l$ for all $j \in D_l$, implying $D_l \subseteq \Delta_l$ and thus $|\Delta_l| \geq |D_l| > d/2$ and $|\Delta_h| < d/2$.

3. Suppose $\bar{w} + \kappa_l + \zeta_l \leq v_l$. There are two cases: $\theta = 0$ and $\theta > 0$.

   Suppose $\theta = 0$. Consider any $(s_A, s_B) \in \mathcal{S}^2$. That is, $(s_{P,j}, s_{-P,j}) \in (\{l,h\} \times \{0,1\})^2$ for all $j \in D$. By Lemma 3.1, $\pi_j(s_{P,j}, s_{-P,j}) = 1/2$ for all $j \in D$. Party $P$'s expected payoff is

$$V(s_P, s_{-P}) = \sum_{j\in D} \pi_j(s_{P,j}, s_{-P,j}) = \frac{1}{2}d.$$

   Consider any $P \in \{A, B\}$ and any $s'_P \in \mathcal{S}$, $s'_P \neq s_P$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s'_{P,k} \neq s_{P,k}$. Moreover, for all $k \in D'$, $s'_{P,k} \in \{l,h\} \times \{0,1\}$ and $s_{-P,k} \in \{l,h\} \times \{0,1\}$. By Lemma 3.1, $\pi_k(s'_{P,k}, s_{-P,k}) = 1/2 = \pi_k(s_{P,k}, s_{-P,k})$ for all $k \in D'$, while $\pi_j(s'_{P,j}, s_{-P,j}) = \pi_j(s_{P,j}, s_{-P,j})$ for all $j \in D \setminus D'$. Therefore, party $P$'s expected payoff is

$$V(s'_P, s_{-P}) = \sum_{j\in D} \pi_j(s'_{P,j}, s_{-P,j}) = \sum_{j\in D} \pi_j(s_{P,j}, s_{-P,j}) = V(s_P, s_{-P}).$$

   Thus, $(s_A, s_B)$ is an equilibrium.

   Suppose $\theta > 0$. Consider any $(s_A^*, s_B^*) \in \mathcal{S}^2$ such that for all $j \in D$, $(s_{A,j}^*, s_{B,j}^*) = ((i_{A,j}, 1), (i_{B,j}, 1))$ for any $(i_{A,j}, i_{B,j}) \in \{l,h\}^2$. By Lemma 3.2, $\pi_j((i_{P,j},1),(i_{-P,j},1)) = 1/2$ for all $j \in D$. Party $P$'s expected payoff is

$$V(s_P^*, s_{-P}^*) = \sum_{j\in D} \pi_j(s_{P,j}^*, s_{-P,j}^*) = \frac{1}{2}d.$$

   Consider any $P \in \{A, B\}$ and any $s'_P \in \mathcal{S}$, $s'_P \neq s_P^*$. There is a nonempty set $D' \subseteq D$ such that $k \in D'$ if and only if $s'_{P,k} \neq s_{P,k}^* = (i_{P,k}, 1)$. Moreover, for all $k \in D'$, $s'_{P,k} \in \{(-i_{P,k}, 1), (i_{P,k}, 0), (-i_{P,k}, 0)\}$, where $-i_{P,k} \in \{l,h\} \setminus \{i_{P,k}\}$, while $s_{-P,k}^* = (i_{-P,k}, 1)$. By Lemma 3.2, $\pi_k(s'_{P,k}, s_{-P,k}^*) = 1 - \pi_k(s_{-P,k}^*, s'_{P,k}) \leq 1 - 1/2 = 1/2$. That is, $\pi_k(s'_{P,k}, s_{-P,k}^*) \leq \pi_k(s_{P,k}^*, s_{-P,k}^*)$ for all $k \in D'$, while $\pi_j(s'_{P,j}, s_{-P,j}^*) = \pi_j(s_{P,j}^*, s_{-P,j}^*)$ for all $j \in D \setminus D'$.

Therefore, party $P$'s expected payoff is

$$V(s'_P, s^*_{-P}) = \sum_{j\in D} \pi_j(s'_{P,j}, s^*_{-P,j}) \leq \sum_{j\in D} \pi_j(s^*_{P,j}, s^*_{-P,j}) = V(s^*_P, s^*_{-P}).$$

Thus, $(s^*_A, s^*_B)$ is an equilibrium.

It follows that for all $\theta \geq 0$, every $(s^*_A, s^*_B) \in \mathcal{S}^2$ such that for all $j \in D$, $(s^*_{A,j}, s^*_{B,j}) = ((i_{A,j}, 1), (i_{B,j}, 1))$ for any $(i_{A,j}, i_{B,j}) \in \{l, h\}^2$ is an equilibrium. Thus, there is an equilibrium $(s^*_A, s^*_B)$ such that $(s^*_{A,j}, s^*_{B,j}) = ((h, 1), (h, 1))$ and $i^*_j = h$ for all $j \in D$, implying $\Delta_h = D$ and thus $|\Delta_h| = d > d/2$.

From 1., 2., and 3. follows that an equilibrium exists for all $v_l \geq w_l$ and $\theta \geq 0$. Suppose $|\Delta_h| > d/2$ in equilibrium. Then it follows from 1., 2., and 3. by contraposition that $v_l \geq \bar{w} + \kappa_l + \zeta_l \geq \bar{w} + \kappa_l - \zeta_l$, which with Observations 1 and 2 implies $\psi_{i,\chi}(0, \delta, j) \geq \psi_{i,\chi}(1, \delta, j)$ for all $i \in \{l, h\}$ and $j \in D$. ■